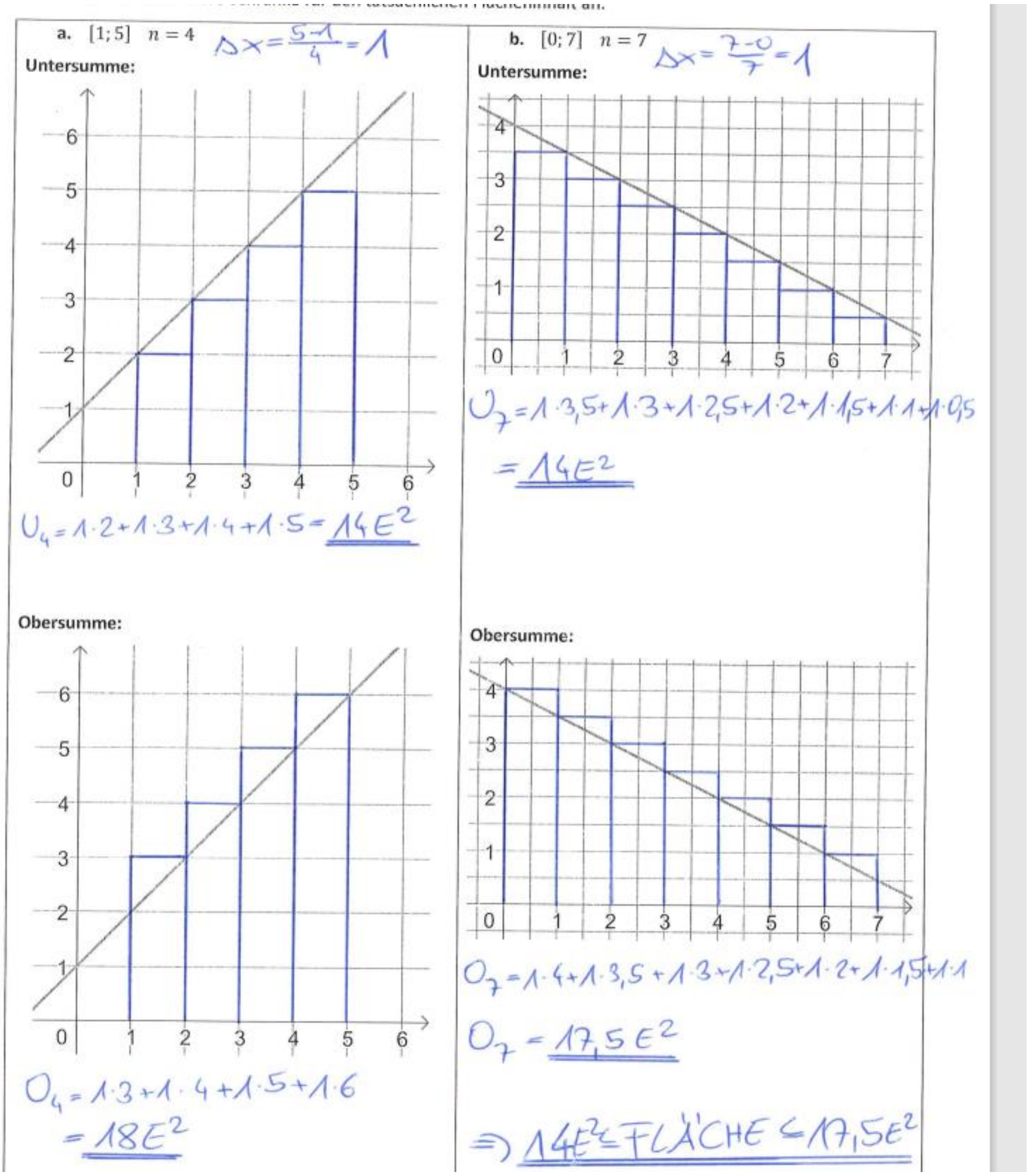
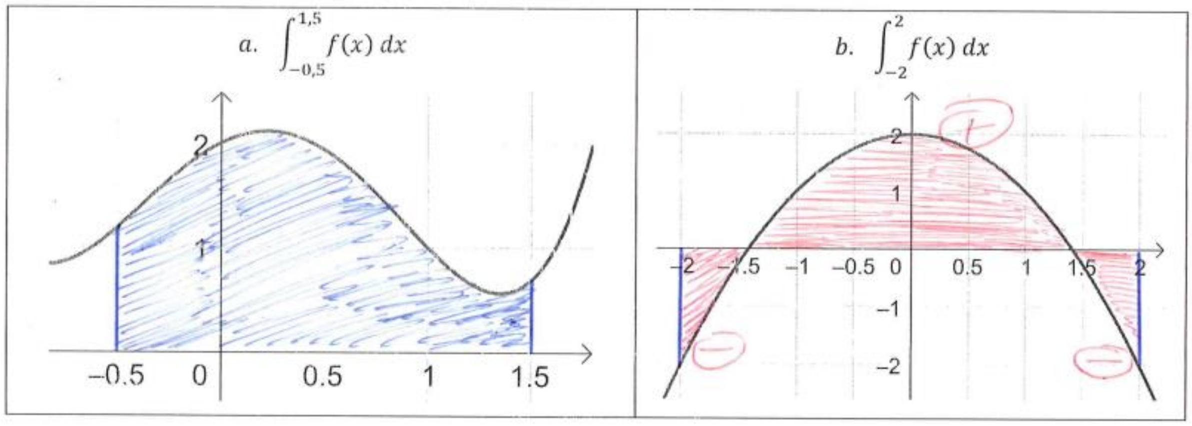
AN4 – Bestimmtes Integral, Flächeninhalt (LÖSUNGEN)

Bsp. 1) Gegeben ist der Graph einer Funktion f. Bestimme näherungsweise die Unter- und Obersumme im gegebenen Intervall durch Unterteilung in n gleich große Teilintervalle. Gib eine untere bzw. obere Schranke für den tatsächlichen Flächeninhalt an.



Bsp. 2)



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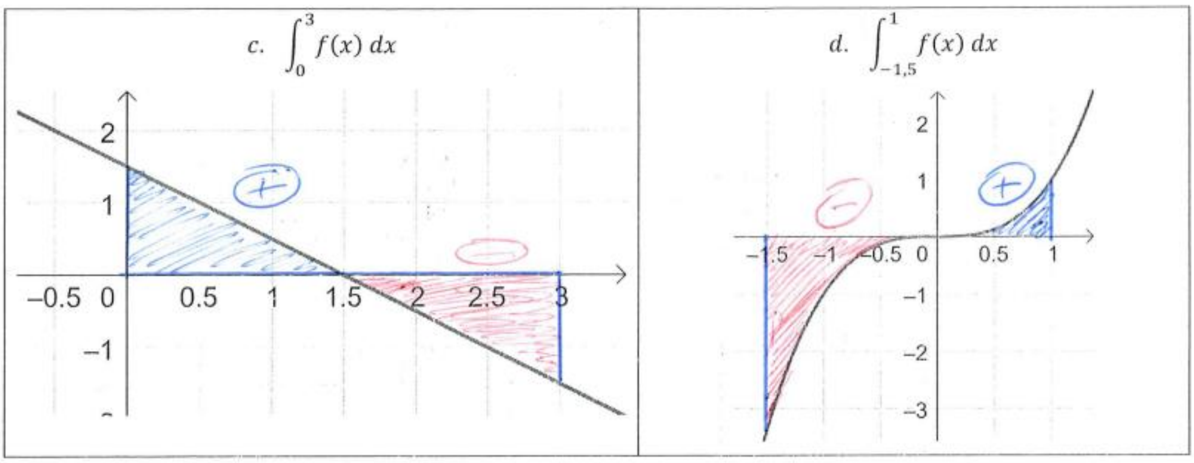
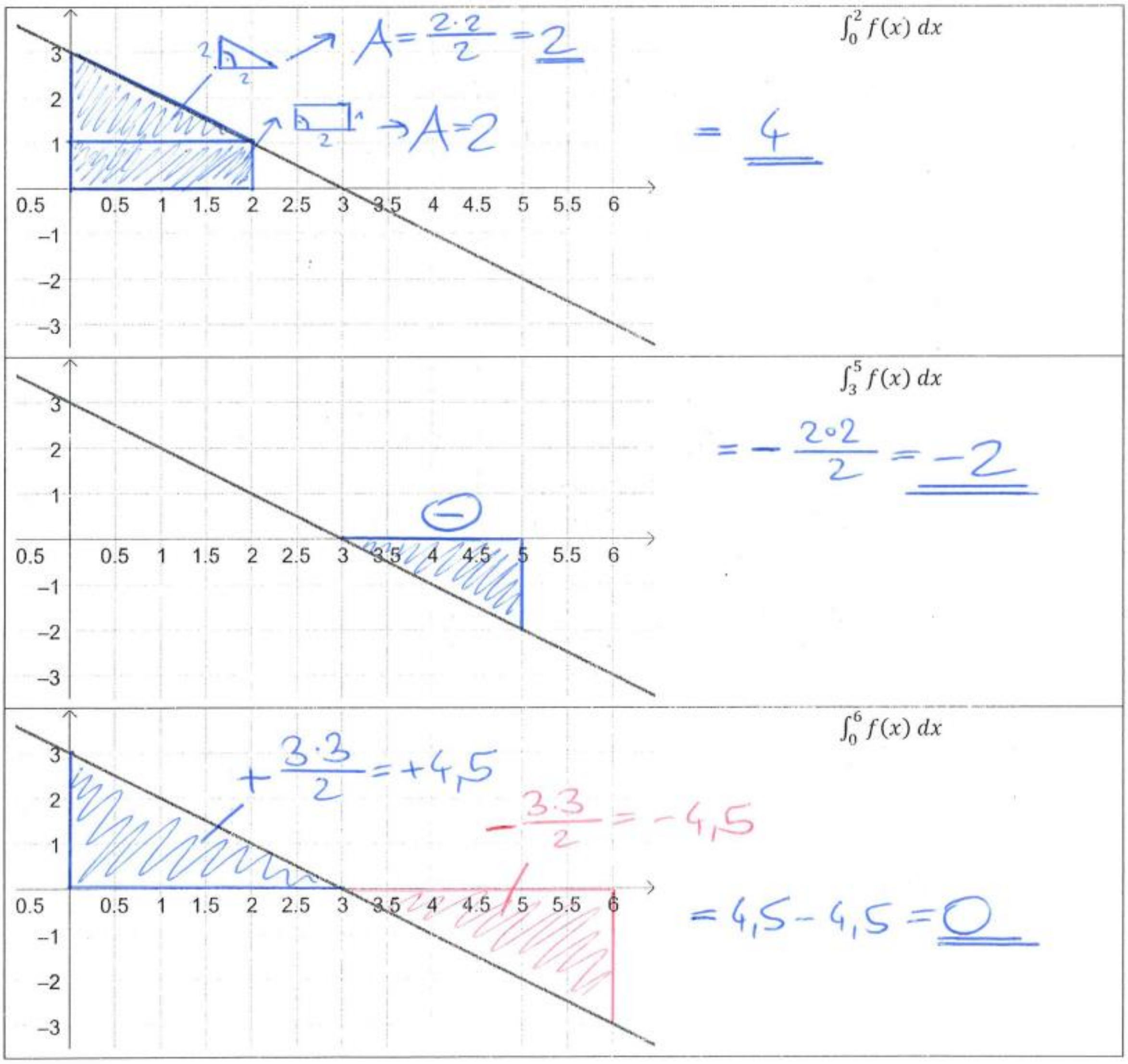
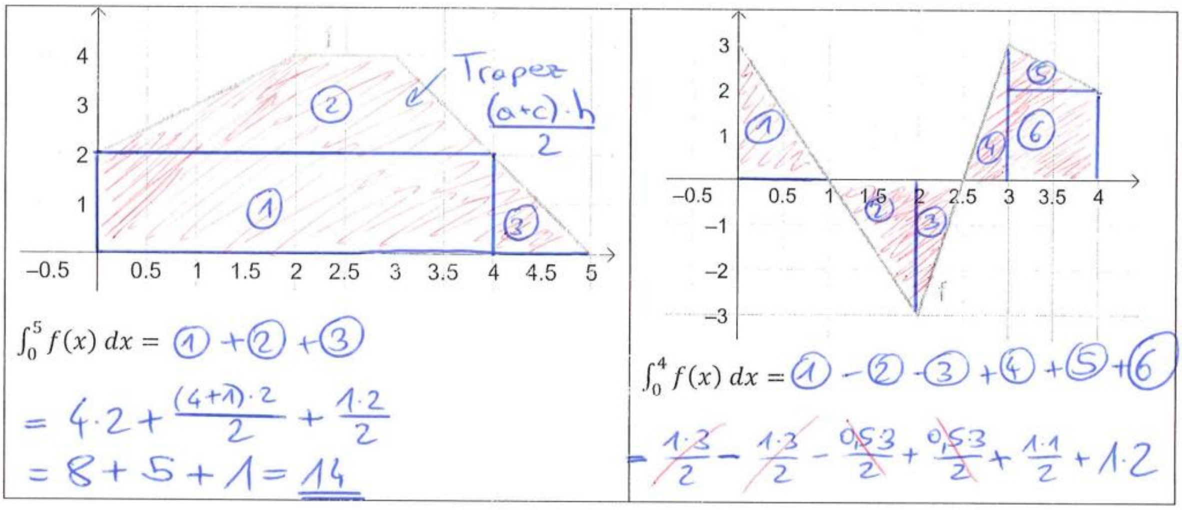


Bsp. 3)



Bsp. 4)



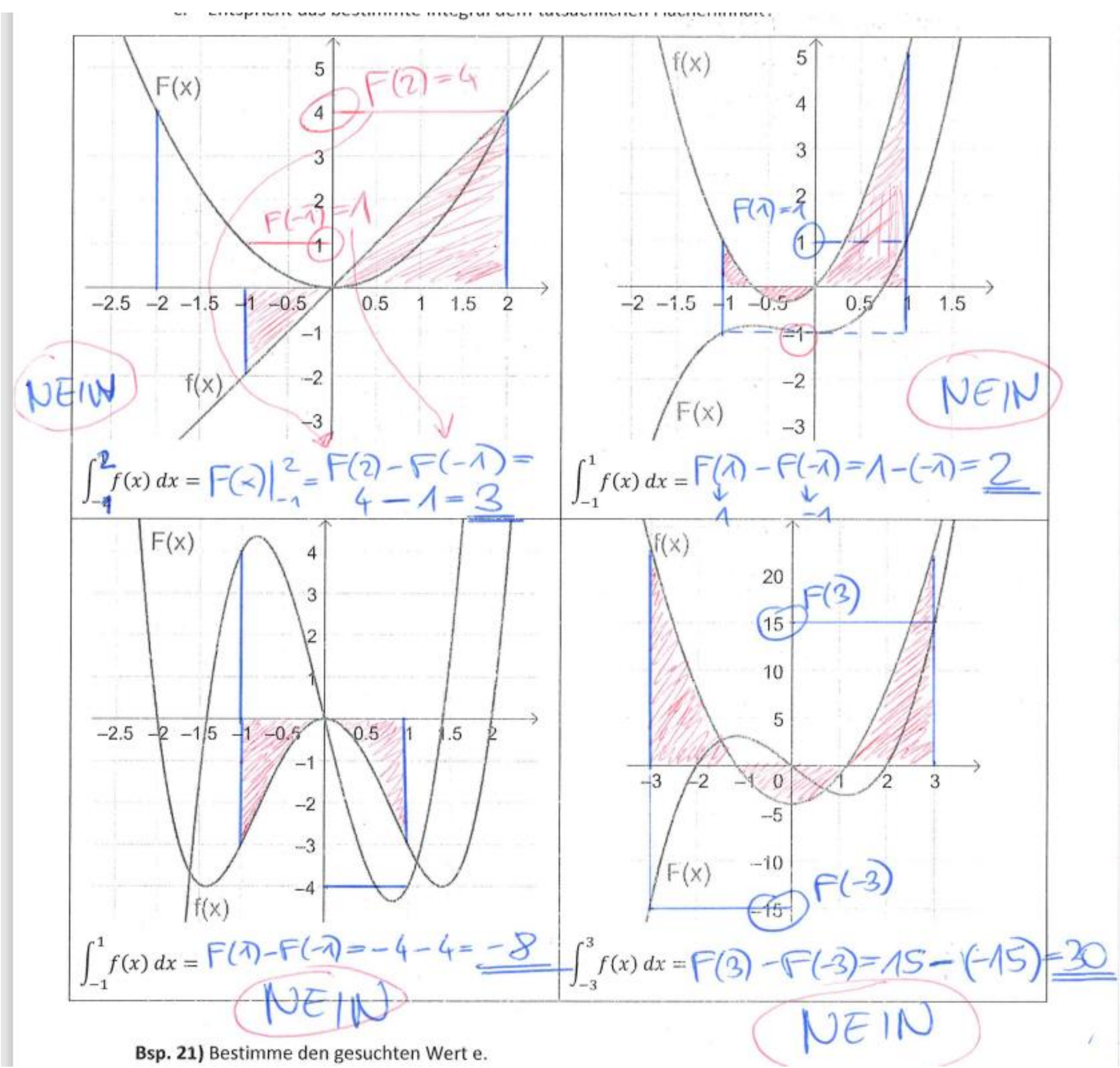
THEORIE: **Integralrechnung: Grundlagen** Seite **18** von **43**

$= 2{,}5$

Bsp. 5)

a. $f(x) = 2x + 1$ $\quad a = 1, b = 3$ $\int_1^3 (2x+1)dx = [x^2+x]_1^3 =$ $9+3-(1+1) = \underline{\underline{10}}$ JA!	**b.** $f(x) = 7$ $\quad a = -2, b = 5$ $\int_{-2}^5 7dx = 7x \Big\|_{-2}^5 =$ $35-(-14) = \underline{\underline{49}}$ JA!
c. $f(x) = -3x + 5$ $\quad a = -2, b = 3$ $\int_{-2}^3 (-3x+5)dx = (-1{,}5x^2+5x)\Big\|_{-2}^3$ $= -13{,}5+15-(-6-10) = \underline{\underline{17{,}5}}$ NEIN!	**d.** $f(x) = 3x^2 + 2x$ $\quad a = 0,\ b = 2$ $\int_0^2 f(x)dx = (x^3+x^2)\Big\|_0^2$ $= 8+4-(0+0) = \underline{\underline{12}}$ JA!
e. $f(x) = x^3 - x$ $\quad a = -4, b = 2$ $\int_{-4}^2 (x^3-x)dx = \left(\frac{x^4}{4}-\frac{x^2}{2}\right)\Big\|_{-4}^2$ $= \frac{16}{4}-\frac{4}{2}-\left(\frac{256}{4}-\frac{16}{2}\right) = 4-2-(56)$ $= \underline{\underline{-54}}$ NEIN!	**f.** $f(x) = 5x^4 + 2x + 4$ $\quad a = 0,\ b = 1$ $\int_0^1 f(x)dx = (x^5+x^2+4x)\Big\|_0^1$ $= 1+1+4-(0+0+0) = \underline{\underline{6}}$ JA!

Bsp. 6)


F(x)
f(x)
F(2) = 4
F(-1) = 1
NEIN
∫₋₁² f(x) dx = F(x)|₋₁² = F(2) − F(−1) = 4 − 1 = 3
f(x)
F(x)
F(1) = 1
NEIN
∫₋₁¹ f(x) dx = F(1) − F(−1) = 1 − (−1) = 2
F(x)
f(x)
∫₋₁¹ f(x) dx = F(1) − F(−1) = −4 − 4 = −8
NEIN
f(x)
F(x)
F(3)
F(−3)
∫₋₃³ f(x) dx = F(3) − F(−3) = 15 − (−15) = 30
NEIN
Bsp. 21) Bestimme den gesuchten Wert e.

Bsp. 7)

a. $\int_1^e (2x+1)\,dx = 10$ $= (x^2+x)\Big\|_1^e = 10$ $e^2+e-(1+1)=10$ $e^2+e-2=10 \quad \| -10$ $e^2+e-12=0$ GG: $(e_1=-4)$ $e_2=3$ $\underline{\underline{e=3}}$	**b.** $\int_e^3 (x^2+x)\,dx = 13{,}5$ $=\left(\frac{x^3}{3}+\frac{x^2}{2}\right)\Big\|_e^3 = 13{,}5$ $\frac{27}{3}+\frac{9}{2}-\frac{e^3}{3}-\frac{e^2}{2}=13{,}5$ GG $\underline{\underline{e_1=-1{,}5}}$ $\underline{\underline{e_2=0}}$

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c. $\int_0^e (2x^3-3x^2)\,dx = 10$ (64) $=\left(2\frac{x^4}{4}-3\frac{x^3}{3}\right)\Big\|_0^e = 64$ $\frac{e^4}{2}-e^3-(0-0)=64$ $\frac{e^4}{2}-e^3=64 \iff (e_1=-2{,}94)$ $e_2=4$	**d.** $\int_e^{10} \frac{1}{x^2}\,dx = 0{,}9$ ($\to x^{-2}$) $=\left(\frac{x^{-1}}{-1}\right)\Big\|_e^{10} = \frac{10^{-1}}{-1}-\frac{e^{-1}}{-1}=0{,}9$ $-0{,}1+\frac{1}{e}=0{,}9$ $\frac{1}{e}=1 \iff \underline{\underline{e=1}}$

Bsp. 8)

a. $\int_{-2}^{2} 2x\,dx + \int_2^3 2x\,dx + \int_3^5 2x\,dx = \int_{-2}^{5} 2x\,dx$ $= x^2\Big\|_{-2}^{5} = 25-4 = \underline{\underline{21}}$	**b.** $\int_1^7 (x^2+2)\,dx - \int_{10}^7 (x^2+2)\,dx =$ $=\int_1^7 (x^2+2)\,dx + \int_7^{10}(x^2+2)\,dx = \int_1^{10}(x^2+2)\,dx$ $=\left(\frac{x^3}{3}+2x\right)\Big\|_1^{10} = \frac{1000}{3}+20-\frac{1}{3}-2=$ $\underline{\underline{351}}$
c. $\int_{-2}^{5} (x^7+x^6)\,dx + \int_5^{-2} (x^7+x^6)\,dx + \int_1^2 x\,dx =$ ~~$\int_{-2}^{5}(x^7+x^6)\,dx - \int_{-2}^{5}(x^7+x^6)\,dx$~~ $+\int_1^2 x\,dx$ $=\int_1^2 x\,dx = \frac{x^2}{2}\Big\|_1^2 = \frac{4}{2}-\frac{1}{2}$ $=2-0{,}5=\underline{\underline{1{,}5}}$	**d.** $\int_2^7 4x+1\,dx + \int_7^8 4x\,dx + \int_7^8 1\,dx =$ $\int_2^7 (4x+1)\,dx + \int_7^8 (4x+1)\,dx =$ $\int_2^8 (4x+1)\,dx = \left(4\cdot\frac{x^2}{2}+x\right)\Big\|_2^8 = (2x^2+x)\Big\|_2^8$ $=2\cdot 64+8-(2\cdot 4+2)=128+8-10=$ $\underline{\underline{126}}$

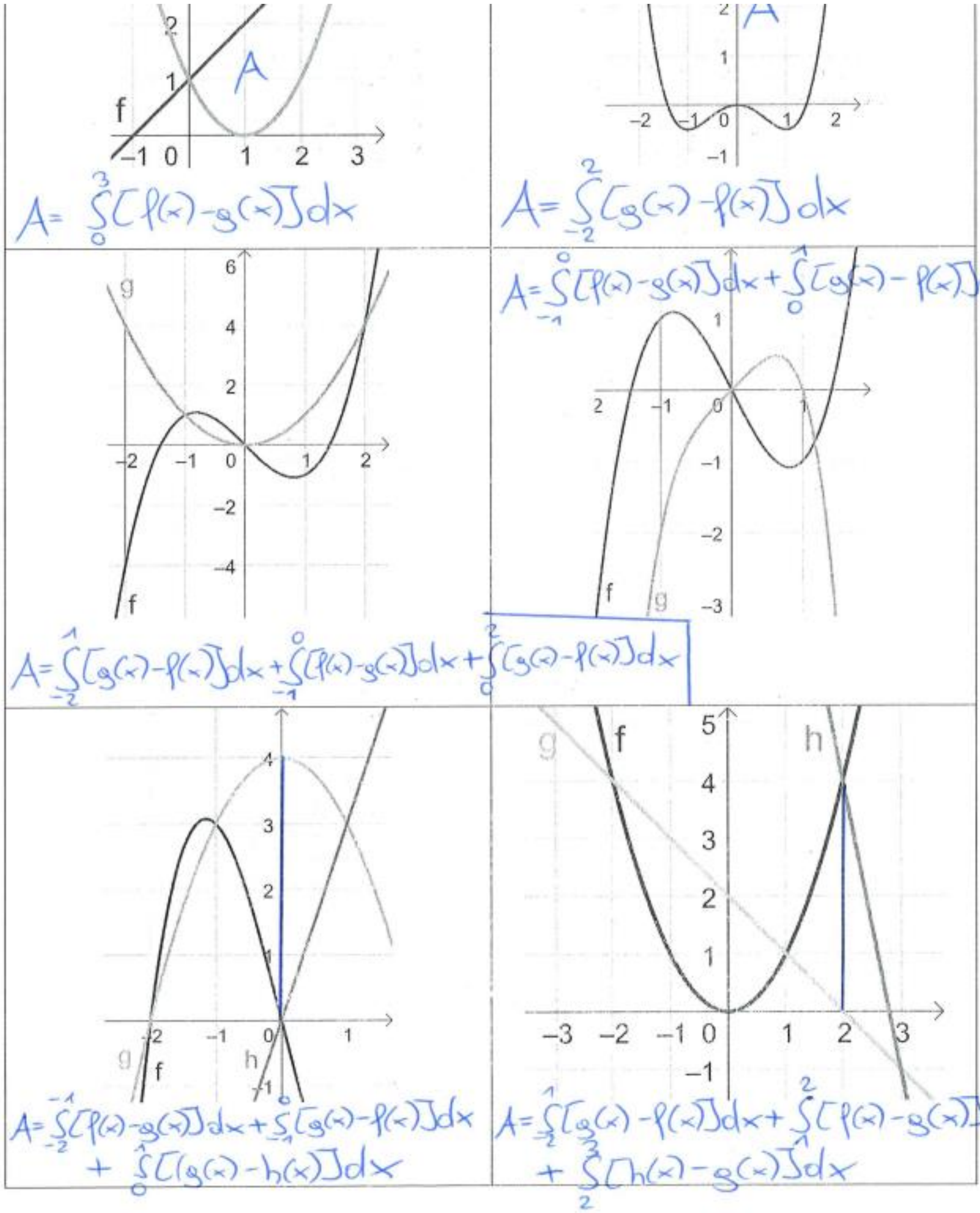
Bsp. 9)

a. $f(x) = 3x^2 - 3$ $[-2; 3]$ $f(x)=0 \Leftrightarrow x_1=-1,\ x_2=1$ $A = \int_{-2}^{-1} f(x)dx - \int_{-1}^{1} f(x)dx + \int_{1}^{3} f(x)dx$ $A = 4-(-4)+20 = 28\,E^2$	**b.** $f(x) = 2x + 4$ $[-6; 2]$ $f(x)=0 \Leftrightarrow x=-2$ $A = -\int_{-6}^{-2} f(x)dx + \int_{-2}^{2} f(x)dx$ $A = -(-16)+16 = 32\,E^2$
c. $f(x) = x^3 - 9x$ $[-5; 4]$ $f(x)=0 \Leftrightarrow x_1=-3,\ x_2=0,\ x_3=3$ $A = -\int_{-5}^{-3} f(x)dx + \int_{-3}^{0} f(x)dx - \int_{0}^{3} f(x)dx + \int_{3}^{4} f(x)dx =$ $A = -(-64)+20{,}25-(-20{,}25)+12{,}25 = 116{,}75\,E^2$	**d.** $f(x) = x^4 - 4x^2$ $[-1; 3]$ $f(x)=0 \Leftrightarrow x_1=-2,\ x_2=0,\ x_3=2$ $A = -\int_{-1}^{0} f(x)dx - \int_{0}^{2} f(x)dx + \int_{2}^{3} f(x)dx$ $A = -(-1{,}13)-(-4{,}27)+16{,}87 = 22{,}27\,E^2$
e. $f(x) = e^x - 2$ $[-3; 5]$ $f(x)=0 \Leftrightarrow x=0{,}69$ $A = -\int_{-3}^{0{,}69} f(x)dx + \int_{0{,}69}^{5} f(x)dx$ $A = 5{,}44 + 137{,}8 = 143{,}24\,E^2$	**f.** $f(x) = \frac{1}{x} - 1$ $[0{,}5; 3]$ $f(x)=0 \Leftrightarrow x=1$ $A = \int_{0{,}5}^{1} f(x)dx - \int_{1}^{3} f(x)dx$ $A = 0{,}19 + 0{,}9 = 1{,}09\,E^2$

Bsp. 10)

a. $f(x) = -x + 3$ $A = 10\,E^2$ (-2) ✓ $8E^2$ $f(x)=0 \Leftrightarrow x=3$ ① $\int_{1}^{3} f(x)dx = 2$ ② $\int_{3}^{e} f(x)dx = -8$ $\left(-\frac{x^2}{2}+3x\right)\Big\vert_3^e = -8$ $\left(-\frac{e^2}{2}+3e\right)-\left(-\frac{9}{2}+9\right) = -8$ GG $(e_1=-1)$ $e_2=7$ $[1;7]$	**b.** $f(x) = -3x^2 + 12$ $A = 37\,E^2$ -5 ✓ $32E^2$ $f(x)=0 \Leftrightarrow (x_1=-2)\ x_2=2$ ① $\int_{1}^{2} f(x)dx = 5$ ② $\int_{2}^{e} f(x)dx = 32$ $-32!$ GG $(e_1=-2)$ $e_2=4$ ✓ $[1;4]$
c. $f(x) = x^3 - 3x^2$ $A = 270\,E^2$ -6 ✓ $264E^2$ $f(x)=0 \Leftrightarrow (x_1=0)\ x_2=3$ ① $\int_{1}^{3} f(x)dx = -6 \Rightarrow (+6)$ ② $\int_{3}^{e} f(x)dx = 264$ $(e_1\approx-4{,}88)$ $e_2=7$ $[1;7]$	**d.** $f(x) = -4x + 16$ $A = 146\,E^2$ -18 ✓ $128E^2$ $f(x)=0 \Leftrightarrow x=4$ ① $\int_{1}^{4} f(x)dx = 18$ ② $\int_{4}^{e} f(x)dx = -128$ $(e_1=-4)$ $e_2=12$ $[1;12]$

Bsp. 11)


$A = \int_0^3 [f(x) - g(x)]\,dx$
$A = \int_{-2}^{2} [g(x) - f(x)]\,dx$
$A = \int_{-1}^{0} [f(x) - g(x)]\,dx + \int_0^1 [g(x) - f(x)]\,dx$
$A = \int_{-2}^{-1} [g(x) - f(x)]\,dx + \int_{-1}^{0} [f(x) - g(x)]\,dx + \int_0^2 [g(x) - f(x)]\,dx$
$A = \int_{-2}^{-1} [f(x) - g(x)]\,dx + \int_{-1}^{0} [g(x) - f(x)]\,dx + \int_0^1 [g(x) - h(x)]\,dx$
$A = \int_{-2}^{1} [g(x) - f(x)]\,dx + \int_1^2 [f(x) - g(x)]\,dx + \int_2^3 [h(x) - g(x)]\,dx$

Bsp. 12)

Bsp. 27) Berechne den Flächeninhalt, der von den Graphen der Funktionen f und g begrenzt wird.

a. $f(x) = x^4 - 3x^2 + 4$, $g(x) = 2x + 4$ ① $f(x)=g(x) \Leftrightarrow x_1=-1, x_2=0; x_3=2$ ② $A = \int_{-1}^{0}[f(x)-g(x)]dx + \int_{0}^{2}[g(x)-f(x)]dx$ $= 0{,}2 + 5{,}6 = \underline{\underline{5{,}8 E^2}}$	**b.** $f(x) = -x$, $g(x) = x^3 - 2x$ ① $f(x)=g(x): x_1=-1; x_2=0; x_3=1$ ② $A = \int_{-1}^{0}[g(x)-f(x)]dx + \int_{0}^{1}[f(x)-g(x)]dx$ $A = 0{,}25 + 0{,}25 = \underline{\underline{0{,}5 E^2}}$
c. $f(x) = x^3 + 4x^2 + 2$, $g(x) = 8 - x$ ① $f(x)=g(x): x_1=-3; x_2=-2; x_3=1$ ② $A = \int_{-3}^{-2}[f(x)-g(x)]dx + \int_{-2}^{1}[g(x)-f(x)]dx$ $A = 0{,}58 + 11{,}25 = \underline{\underline{11{,}83 E^2}}$	**d.** $f(x) = x^3 - 4x$, $g(x) = 4 - x^2$ ① $f(x)=g(x): x_1=-2; x_2=-1; x_3=2$ ② $A = \int_{-2}^{-1}[f(x)-g(x)]dx + \int_{-1}^{2}[g(x)-f(x)]dx$ $A = 0{,}58 + 11{,}25 = \underline{\underline{11{,}83 E^2}}$