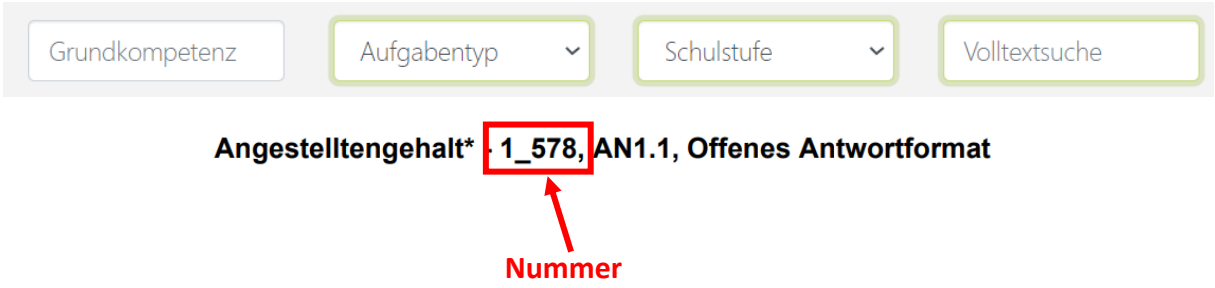
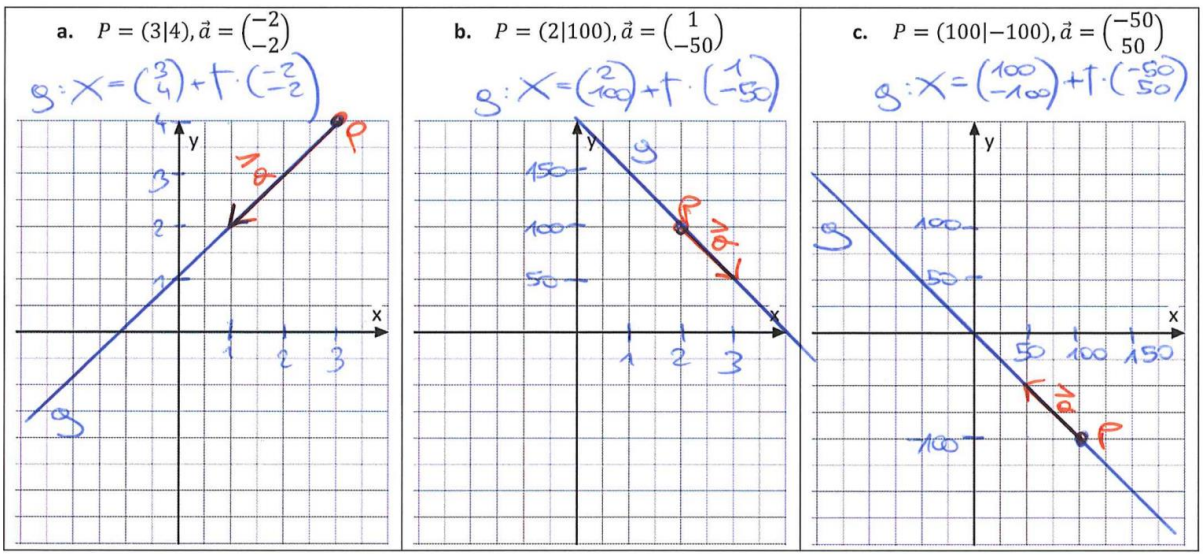
AG3.4 Geraden (Lösungen)

Lösungen Maturaaufgaben:

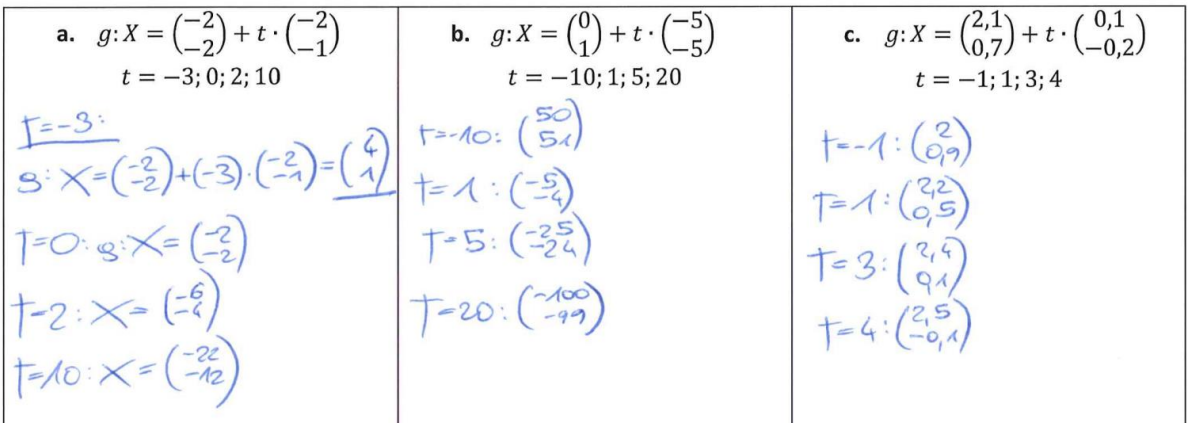
1) Gehe zum Aufgabenpool Mathematik AHS: https://prod.aufgabenpool.at/amn/index.php?id=M
2) Gib im Feld „**Volltextsuche**“ die **Nummer** ein. Du kommst zur zugehörigen Aufgabe. Die Lösungen sind bei der Aufgabe enthalten.



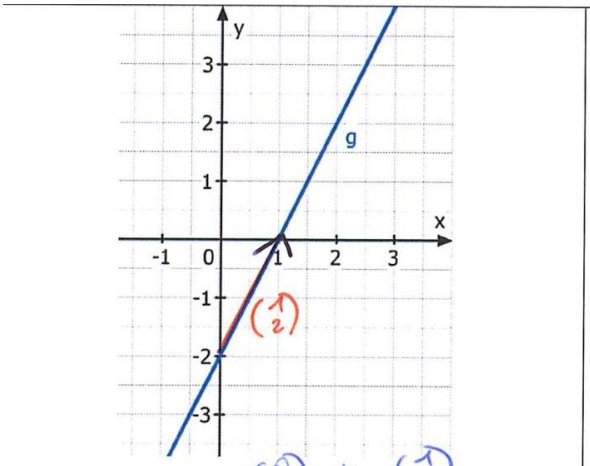
Bsp. 1)



Bsp. 2)

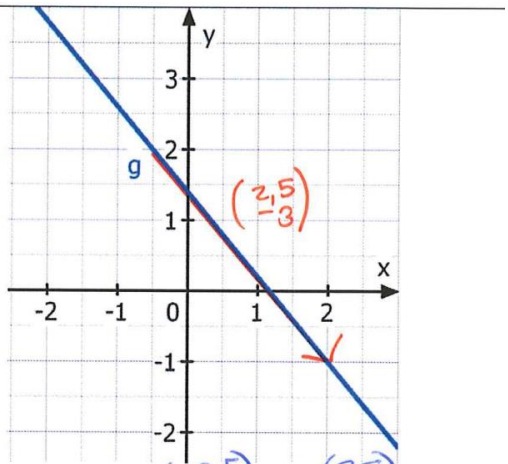


Bsp. 3)



$g_1: X = \begin{pmatrix} 0 \\ -2 \end{pmatrix} + t_1 \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$g_2: X = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t_2 \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$



$g_1: X = \begin{pmatrix} -0{,}5 \\ 2 \end{pmatrix} + t_1 \cdot \begin{pmatrix} 2{,}5 \\ -3 \end{pmatrix}$

$g_2: X = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + t_2 \cdot \begin{pmatrix} 5 \\ -6 \end{pmatrix}$

Bsp. 4)

a. $P = (3|4|1), \vec{a} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$

$g: X = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$

① $t=1: X = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + 1 \cdot \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \\ -1 \end{pmatrix}$

② $t=-2: X = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + (-2) \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix}$

③ $t=10: X = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + 10 \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -7 \\ 34 \\ -19 \end{pmatrix}$

Werte für t können beliebig gewählt werden!

b. $P = (-1|7|4), \vec{a} = \begin{pmatrix} -4 \\ 12 \\ 2 \end{pmatrix}$

$g: X = \begin{pmatrix} -1 \\ 7 \\ 4 \end{pmatrix} + t \cdot \begin{pmatrix} -4 \\ 12 \\ 2 \end{pmatrix}$

① $t=2: \begin{pmatrix} -1 \\ 7 \\ 4 \end{pmatrix} + 2 \cdot \begin{pmatrix} -4 \\ 12 \\ 2 \end{pmatrix} = \begin{pmatrix} -9 \\ 31 \\ 8 \end{pmatrix}$

② $t=-5: \begin{pmatrix} -1 \\ 7 \\ 4 \end{pmatrix} + (-5) \begin{pmatrix} -4 \\ 12 \\ 2 \end{pmatrix} = \begin{pmatrix} 19 \\ -53 \\ -6 \end{pmatrix}$

③ $t=-1: \begin{pmatrix} -1 \\ 7 \\ 4 \end{pmatrix} + (-1) \cdot \begin{pmatrix} -4 \\ 12 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$

c. $P = (13|-4|11), \vec{a} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$

$g: X = \begin{pmatrix} 13 \\ -4 \\ 11 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$

① $t=10: \begin{pmatrix} 13 \\ -4 \\ 11 \end{pmatrix} + 10 \cdot \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -14 \\ 21 \end{pmatrix}$

② $t=-20: \begin{pmatrix} 13 \\ -4 \\ 11 \end{pmatrix} + (-20) \cdot \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 33 \\ 16 \\ -9 \end{pmatrix}$

③ $t=5: \begin{pmatrix} 13 \\ -4 \\ 11 \end{pmatrix} + 5 \cdot \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ -9 \\ 16 \end{pmatrix}$

Bsp. 5)

a. $g: X = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} -5 \\ 3 \end{pmatrix} \quad - \quad P = (x|7)$

I / II $\begin{pmatrix} x \\ 7 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} -5 \\ 3 \end{pmatrix}$

II $7 = 1 + 3t \quad | -1$

$6 = 3t \quad | :3$

$t = 2$

I $\rightsquigarrow x = 0 - 5t = \underline{\underline{-10}}$

b. $g: X = \begin{pmatrix} -3 \\ 6 \end{pmatrix} + t \cdot \begin{pmatrix} -2 \\ -1 \end{pmatrix} \quad - \quad P = (1|y)$

I $\begin{pmatrix} 1 \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \end{pmatrix} + t \cdot \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

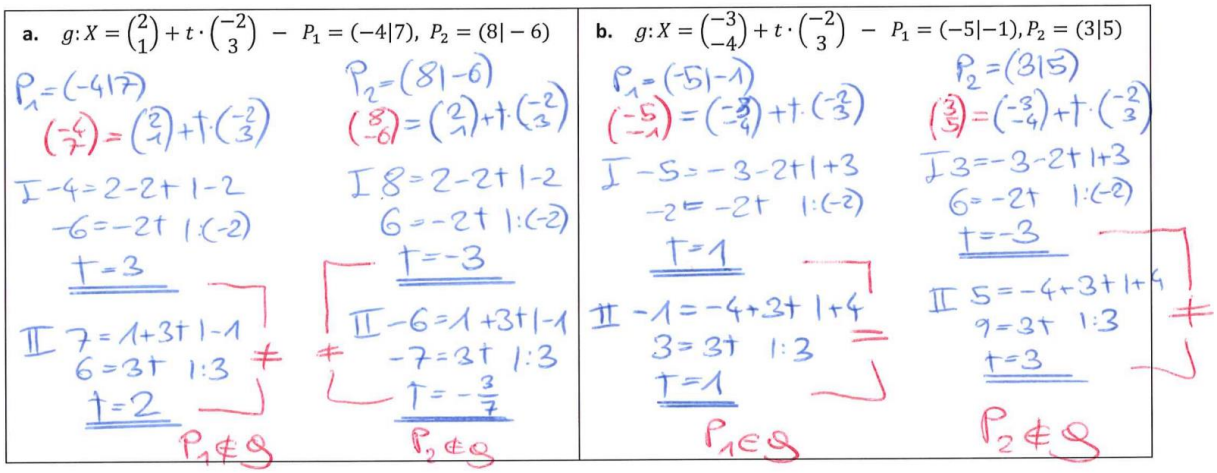
I $1 = -3 - 2t \quad | +3$

$4 = -2t \quad | :(-2)$

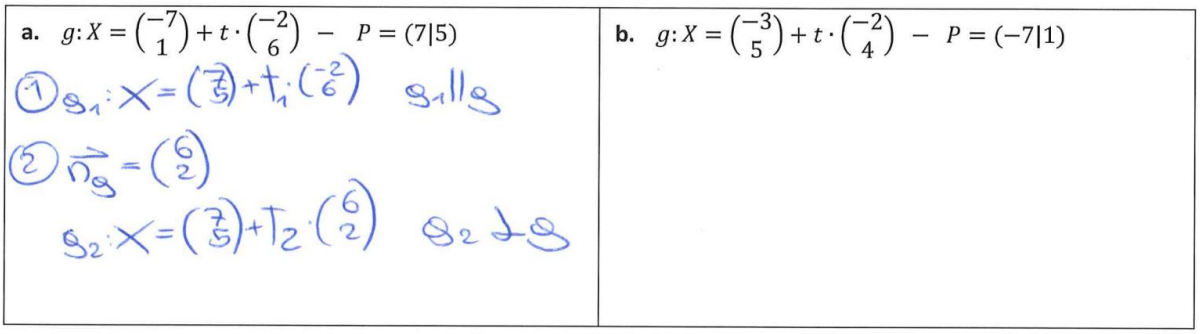
$\underline{\underline{t = -2}}$

$\rightsquigarrow y = 6 - t = 6 - (-2) = \underline{\underline{8}}$

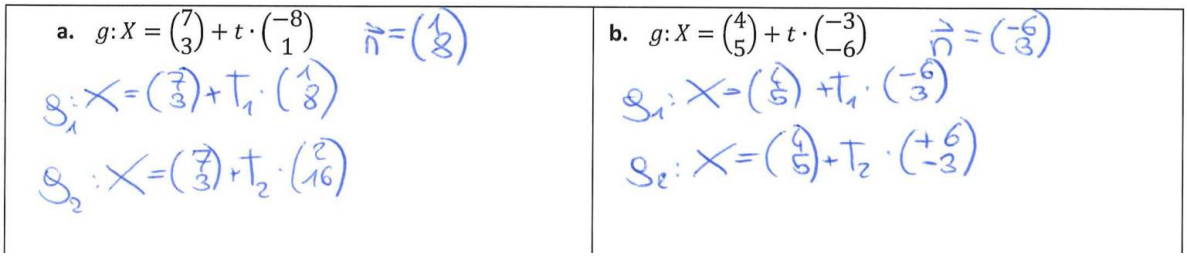
Bsp. 6)



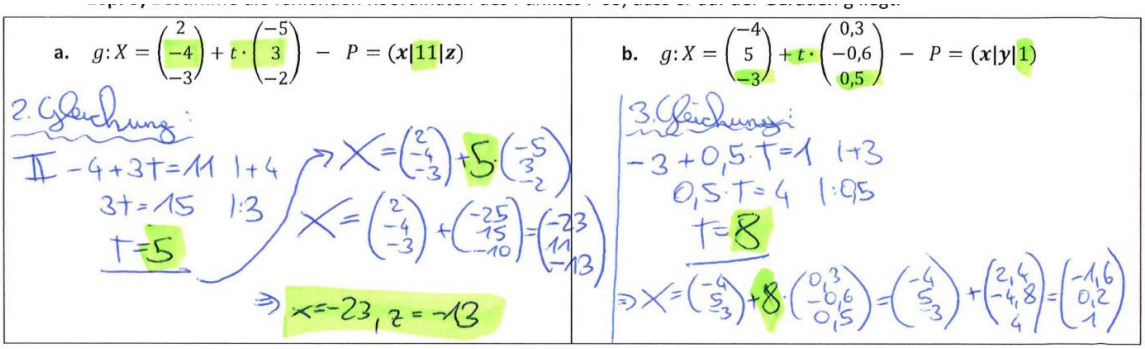
Bsp. 7)



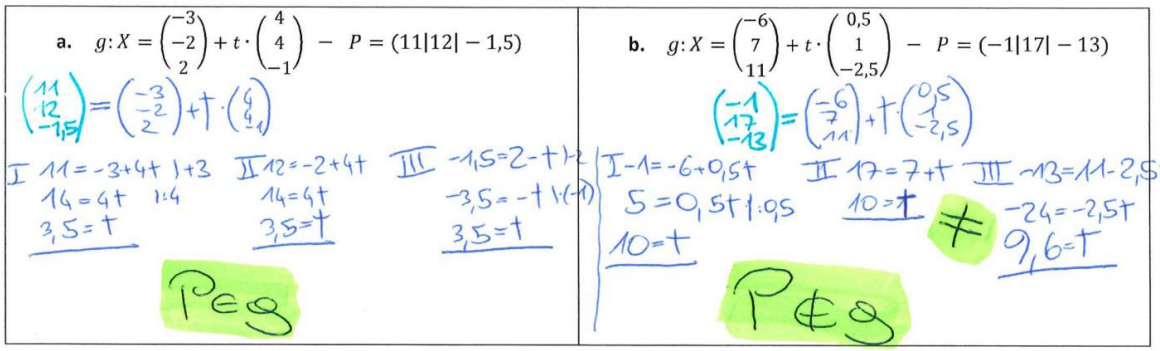
Bsp. 8)



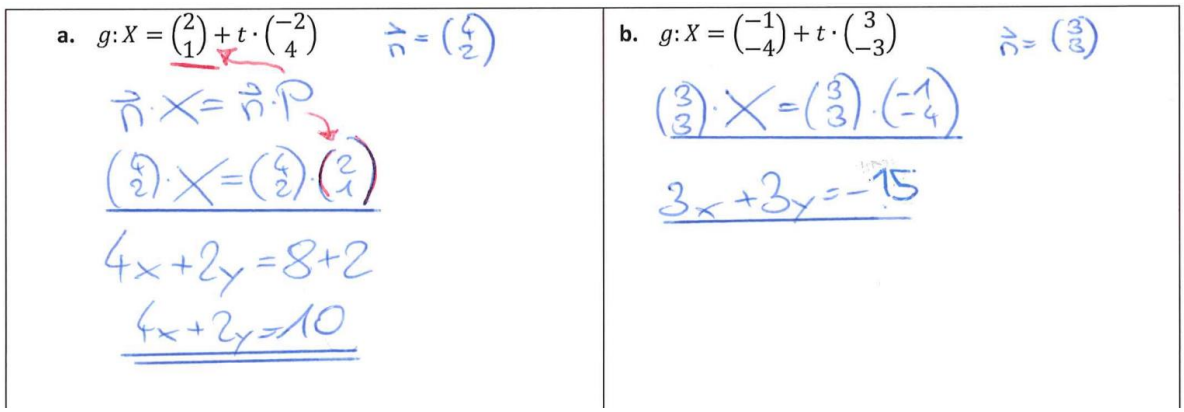
Bsp. 9)



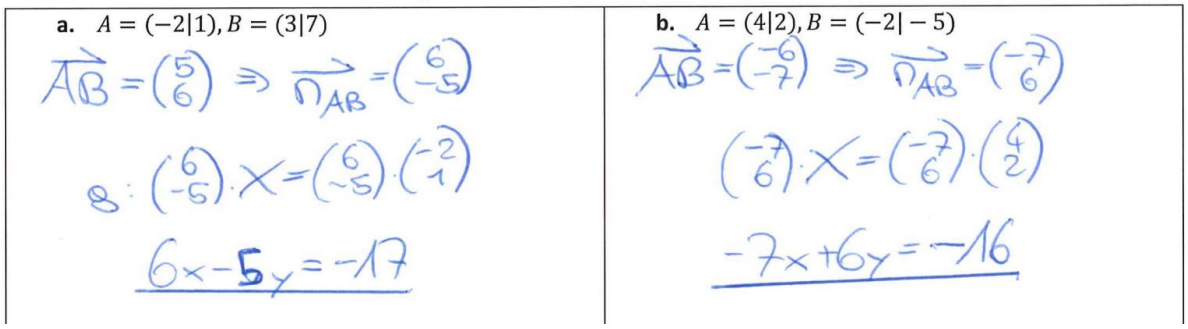
Bsp. 10)



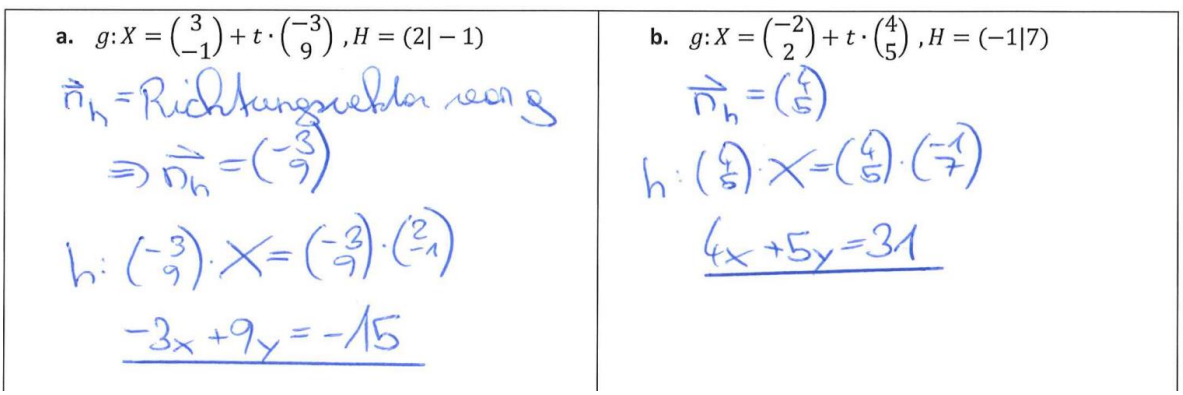
Bsp. 11)



Bsp. 12)



Bsp. 13)



Bsp. 14)

a. $g: X = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} -5 \\ 7 \end{pmatrix}$

$\vec{RV}_g = \begin{pmatrix} -5 \\ 7 \end{pmatrix}$

$\vec{n}_g = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$

b. $g: 3x - 5y = 1$

$\vec{n}_g = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$

$\vec{RV}_g = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$

Bsp. 15)

a. $g: 2x + 4y = 10$

$2x = 10 - 4y \mid :2$

$x = 5 - 2y$

$y = 1 \Rightarrow x = 3 \quad P_1 = (3|1)$

$y = 3 \Rightarrow x = -1 \quad P_2 = (-1|3)$

$y = 10 \Rightarrow x = -15 \quad P_3 = (-15|10)$

b. $g: -3x - y = -4 \quad \mid +3x$

$-y = 3x - 4 \mid \cdot(-1)$

$y = -3x + 4$

$x = 1 \Rightarrow y = 1 \quad P_1 = (1|1)$

$x = -2 \Rightarrow y = 10 \quad P_2 = (-2|10)$

$x = 0 \Rightarrow y = 4 \quad P_3 = (0|4)$

Bsp. 16)

a. $g: 3x - 2y = 10 \quad - \quad P_1 = (2|-2), P_2 = (-1|-3)$

P_1: $x = 2, y = -2$:

$3 \cdot 2 - 2 \cdot (-2) = 10$

$6 + 4 = 10$

$10 = 10$ ✓ $\quad P_1 \in g$

P_2: $3 \cdot (-1) - 2 \cdot (-3) = 10$

$-3 + 6 = 10$

$3 \neq 10 \quad P_2 \notin g$

b. $g: -5x + 4y = -1 \quad - \quad P_1 = (1|-1), P_2 = (1|1)$

P_1: $-5 + 4 \cdot (-1) = -1$

$-5 - 4 = -1 \quad P_1 \notin g$

$-9 \neq -1$

P_2: $-5 + 4 = -1$

$-1 = -1 \quad P_2 \in g$

Bsp. 17)

a. $g: 5x + y = 15$

① $\vec{n}_g = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \Rightarrow \vec{RV}_g = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

② $P \in g: x = 3, y = 0 \quad P = (3|0)$

$g: X = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

b. $g: -2x + 6y = 14$

① $\vec{n}_g = \begin{pmatrix} -2 \\ 6 \end{pmatrix} \Rightarrow \vec{RV}_g = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

② $P \in g: x = -1, y = 2 \quad P = (-1|2)$

$g: X = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

Bsp. 18)

a. $g: X = \begin{pmatrix} -2 \\ 4 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

$\vec{n}_g = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

$-x + y = 6$

b. $g: X = \begin{pmatrix} 9 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

$\vec{n}_g = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 2 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 9 \\ 2 \end{pmatrix}$

$2x + 6y = 30$

Bsp. 19)

a. $g: y = -2x + 7$

① $k = -2 \Rightarrow \vec{RV}_g = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

② $Pe_g = \begin{pmatrix} 0 \\ 7 \end{pmatrix}$

$g: X = \begin{pmatrix} 0 \\ 7 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

b. $g: y = 7x + 12$

① $\vec{RV}_g = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$

② $Pe_g = \begin{pmatrix} 0 \\ 12 \end{pmatrix}$

$g: X = \begin{pmatrix} 0 \\ 12 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 7 \end{pmatrix}$

Bsp. 20)

a. $g: A = (-7|9), B = (-5|5)$

① $\vec{AB} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$g: X = \begin{pmatrix} -7 \\ 9 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

② $\vec{n}_g = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -7 \\ 9 \end{pmatrix}$

③ $2x + y = -5$

④ $y = -2x - 5$

b. $g: A = (3|1), B = (10|-6)$

① $\vec{AB} = \begin{pmatrix} 7 \\ -7 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$g: X = \begin{pmatrix} 3 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

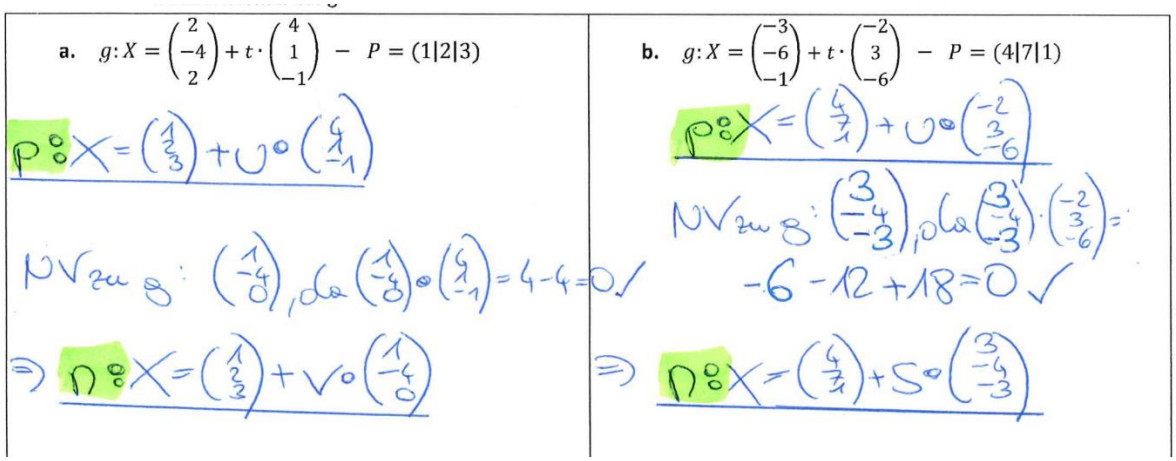
② $\vec{n}_g = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

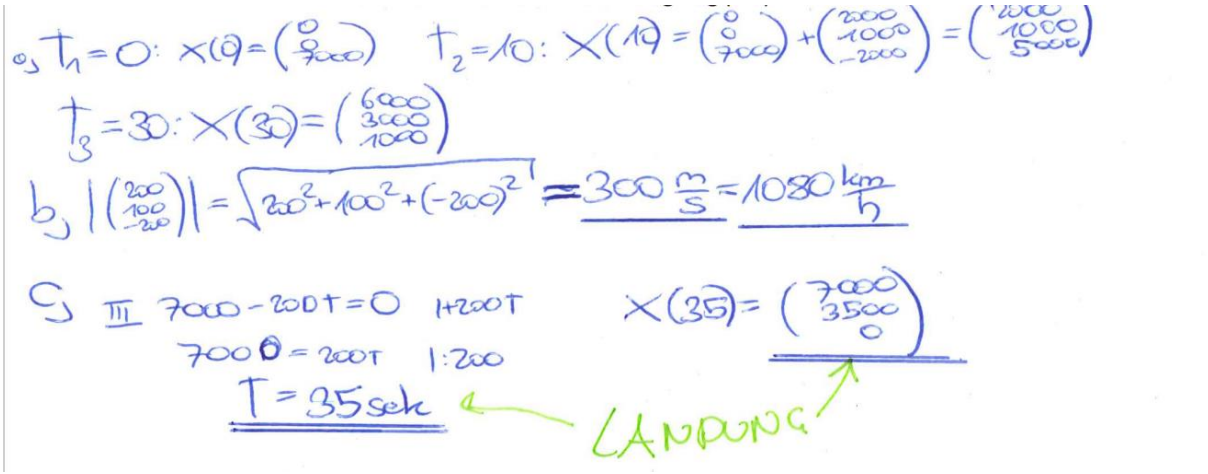
③ $x + y = 4$

④ $y = -x + 4$

Bsp. 21)



Bsp. 22)



Bsp. 23)