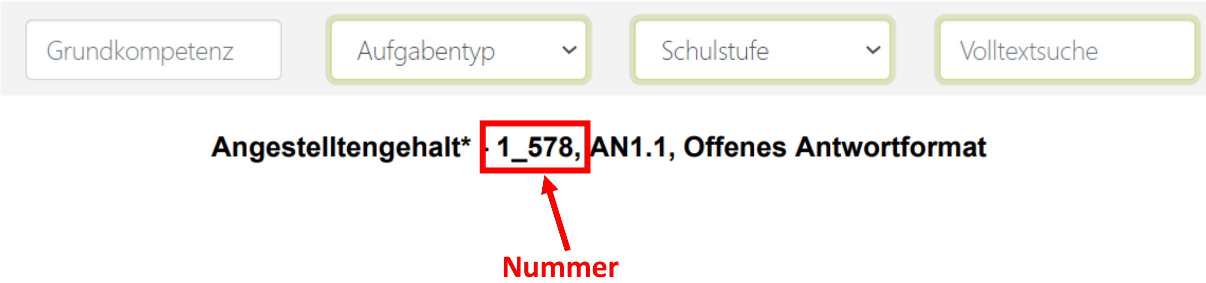
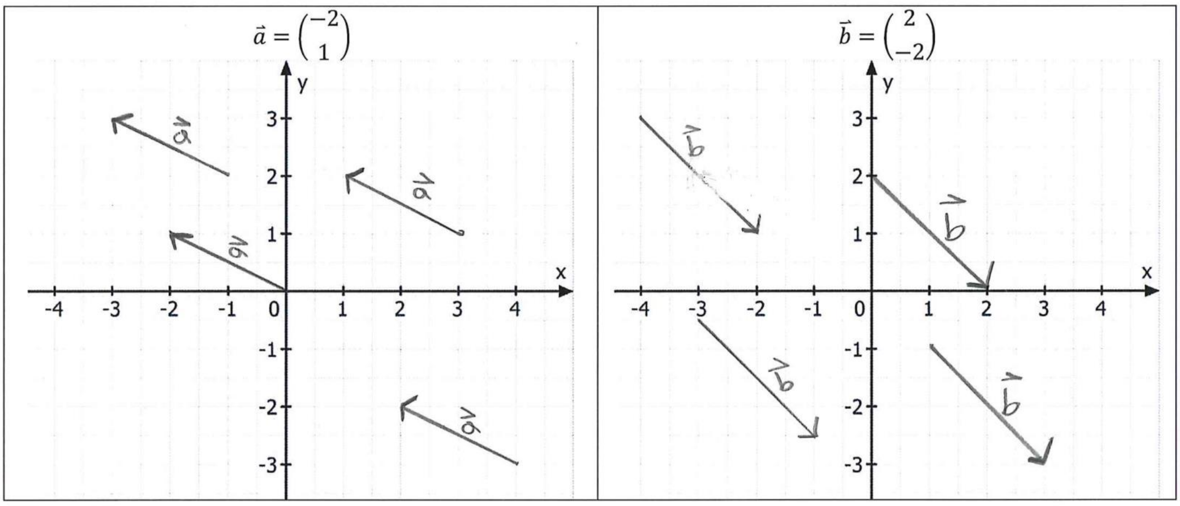
AG3 Vektoren (Lösungen)

Lösungen Maturaaufgaben:

1) Gehe zum Aufgabenpool Mathematik AHS: https://prod.aufgabenpool.at/amn/index.php?id=M
2) Gib im Feld „**Volltextsuche**“ die **Nummer** ein. Du kommst zur zugehörigen Aufgabe. Die Lösungen sind bei der Aufgabe enthalten.



Bsp. 1)



Bsp. 2)

$\vec{a} = \binom{-2}{5} \rightarrow -\vec{a} = \binom{2}{-5}$	$\vec{b} = \binom{12}{-3} \rightarrow -\vec{b} = \binom{-12}{3}$	$\vec{c} = \binom{-14}{-14} \rightarrow -\vec{c} = \binom{14}{14}$
$\vec{d} = \binom{8}{9} \rightarrow -\vec{d} = \binom{-8}{-9}$	$\vec{e} = \binom{-4}{-4} \rightarrow -\vec{e} = \binom{4}{4}$	$\vec{f} = \binom{102}{-103} \rightarrow -\vec{f} = \binom{-102}{103}$

Bsp. 3)

$\vec{a} + \vec{b} = \binom{3}{2} + \binom{-3}{-2} = \binom{0}{0}$	$\vec{a} - \vec{c} = \binom{3}{2} - \binom{13}{-12}$	$\vec{b} + \vec{e} = \binom{-7}{-6}$
$\vec{d} - \vec{a} = \binom{-7}{6} - \binom{3}{2} = \binom{-10}{4}$	$\vec{b} - \vec{d} = \binom{-3}{-2} - \binom{-7}{6} = \binom{4}{-8}$	$\vec{c} - \vec{d} = \binom{20}{-18}$
$\vec{e} - \vec{a} = \binom{-4}{-4} - \binom{3}{2} = \binom{-7}{-6}$	$\vec{c} + \vec{a} = \binom{16}{-10}$	$\vec{b} - \vec{a} = \binom{-6}{-4}$

Bsp. 4)

$\vec{a}+\vec{b}=\begin{pmatrix}1\\2\\3\end{pmatrix}+\begin{pmatrix}-1\\-5\\-3\end{pmatrix}=\begin{pmatrix}0\\-3\\0\end{pmatrix}$	$\vec{a}-\vec{c}=\begin{pmatrix}1\\2\\3\end{pmatrix}-\begin{pmatrix}3\\7\\6\end{pmatrix}=\begin{pmatrix}-2\\-5\\-3\end{pmatrix}$	$\vec{b}+\vec{e}=\begin{pmatrix}9\\5\\7\end{pmatrix}$
$\vec{d}-\vec{a}=\begin{pmatrix}-12\\20\\-16\end{pmatrix}$	$\vec{b}-\vec{d}=\begin{pmatrix}10\\-27\\10\end{pmatrix}$	$\vec{c}-\vec{d}=\begin{pmatrix}14\\-15\\19\end{pmatrix}$
$\vec{e}-\vec{a}=\begin{pmatrix}9\\8\\7\end{pmatrix}$	$\vec{c}+\vec{a}=\begin{pmatrix}4\\9\\9\end{pmatrix}$	$\vec{b}-\vec{a}=\begin{pmatrix}-2\\-7\\-6\end{pmatrix}$

Bsp. 5)

$2\cdot\vec{a}=2\circ\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}6\\4\end{pmatrix}$	$-3\cdot\vec{a}=\begin{pmatrix}-9\\-4\end{pmatrix}$	$10\cdot\vec{a}=\begin{pmatrix}30\\20\end{pmatrix}$
$4\cdot\vec{b}=4\cdot\begin{pmatrix}-1\\-5\\-3\\2\end{pmatrix}=\begin{pmatrix}-4\\-20\\-12\\8\end{pmatrix}$	$-0{,}5\cdot\vec{b}=\begin{pmatrix}0{,}5\\2{,}5\\1{,}5\\-1\end{pmatrix}$	$100\cdot\vec{b}=\begin{pmatrix}-100\\-500\\-300\\200\end{pmatrix}$
$0{,}1\cdot\vec{c}=0{,}1\cdot\begin{pmatrix}3\\7\\6\end{pmatrix}=\begin{pmatrix}0{,}3\\0{,}7\\0{,}6\end{pmatrix}$	$1{,}5\cdot\vec{c}=\begin{pmatrix}4{,}5\\10{,}5\\9\end{pmatrix}$	$5\cdot\vec{c}=\begin{pmatrix}15\\35\\30\end{pmatrix}$

Bsp. 6)

$\vec{a}\cdot\vec{b}=\begin{pmatrix}3\\2\end{pmatrix}\cdot\begin{pmatrix}-3\\-2\end{pmatrix}=-9-4=\underline{-13}$	$\vec{a}\cdot\vec{c}=3\cdot13+2\cdot(-12)=$ $39-24=\underline{15}$	$\vec{b}\cdot\vec{e}=(-3)\cdot(-4)+(-2)\cdot(-4)=$ $=12+8=\underline{20}$
$\vec{d}\cdot\vec{a}=$ $\begin{pmatrix}-7\\6\end{pmatrix}\cdot\begin{pmatrix}3\\2\end{pmatrix}=-21+12=\underline{-9}$	$\vec{b}\cdot\vec{d}=(-3)\cdot(-7)+(-2)\cdot6=$ $=21-12=\underline{9}$	$\vec{c}\cdot\vec{d}=$ $13\cdot(-7)+(-12)\cdot(6)=$ $=-91-72=\underline{-163}$

Bsp. 7)

$\vec{a}\cdot\vec{b}=2-10+18+4=\underline{14}$

Bsp. 8)

$\overrightarrow{AB} = B - A = \begin{pmatrix}-3\\-2\end{pmatrix} - \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}-6\\-4\end{pmatrix}$	$\overrightarrow{AD} = D - A = \begin{pmatrix}-10\\4\end{pmatrix}$	$\overrightarrow{EC} = C - E = \begin{pmatrix}17\\-8\end{pmatrix}$
$\overrightarrow{CD} = D - C = \begin{pmatrix}-20\\18\end{pmatrix}$	$\overrightarrow{BE} = E - B = \begin{pmatrix}-1\\-2\end{pmatrix}$	$\overrightarrow{DB} = B - D = \begin{pmatrix}4\\-8\end{pmatrix}$

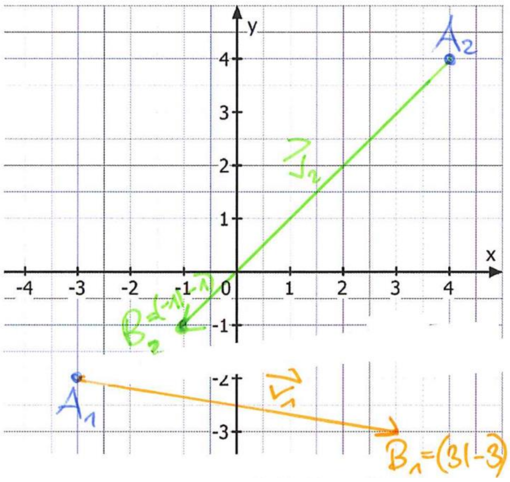
Bsp. 9)

$\lvert\overrightarrow{AB}\rvert = \overrightarrow{AB} = B - A = \begin{pmatrix}1\\4\end{pmatrix}$ $\Rightarrow \lvert\overrightarrow{AB}\rvert = \sqrt{1^2+4^2} \approx \underline{\underline{4,1}}$	$\lvert\overrightarrow{AD}\rvert =$ ① $\overrightarrow{AD} = D - A = \begin{pmatrix}-1\\-4\end{pmatrix}$ ② $\lvert\overrightarrow{AD}\rvert = \sqrt{(-1)^2+(-4)^2} = \sqrt{17}$ $\approx \underline{\underline{4,1}}$	$\lvert\overrightarrow{EC}\rvert =$ ① $\overrightarrow{EC} = C - E = \begin{pmatrix}-15\\-21\end{pmatrix}$ ② $\lvert\overrightarrow{CE}\rvert = \sqrt{(-15)^2+(-21)^2}$ $\approx \underline{\underline{25,8}}$
$\lvert\overrightarrow{CD}\rvert =$ ① $\overrightarrow{CD} = D - C = \begin{pmatrix}2\\-13\end{pmatrix}$ ② $\lvert\overrightarrow{CD}\rvert = \sqrt{2^2+(-13)^2} \approx \underline{\underline{13,2}}$	$\lvert\overrightarrow{BE}\rvert =$ ① $\overrightarrow{BE} = \begin{pmatrix}-19\\-16\end{pmatrix}$ ② $\lvert\overrightarrow{BE}\rvert \approx \underline{\underline{24,8}}$	$\lvert\overrightarrow{DB}\rvert =$ ① $\overrightarrow{DB} = \begin{pmatrix}3\\8\end{pmatrix}$ ② $\lvert\overrightarrow{DB}\rvert = \sqrt{4+64} \approx \underline{\underline{8,2}}$

Bsp. 10)

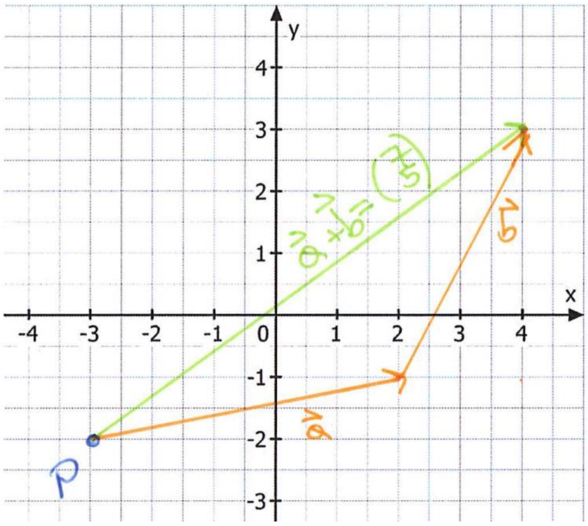
Bsp. 11) Addiere die Vektoren geometrisch als **Addition** von **Punkt** und **Pfeil.** Gib die Koordinaten des Endpunktes an B_1 bzw. B_2 an. Kontrolliere rechnerisch.

a. $A_1 = (-3\mid-2), \vec{v}_1 = \binom{6}{-1}$ $A_1 + \vec{v}_1 =$ $\begin{pmatrix}-3\\-2\end{pmatrix} + \begin{pmatrix}6\\-1\end{pmatrix} = \underline{\underline{\begin{pmatrix}3\\-3\end{pmatrix}}}$	b. $A_2 = (4\mid4), \vec{v}_2 = \binom{-5}{-5}$ $A_2 + \vec{v}_2 =$ $\begin{pmatrix}4\\4\end{pmatrix} + \begin{pmatrix}-5\\-5\end{pmatrix} = \underline{\underline{\begin{pmatrix}-1\\-1\end{pmatrix}}}$

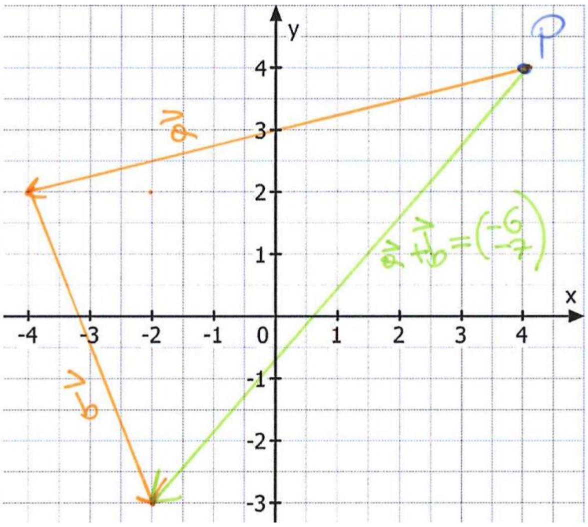


Bsp. 11)

$P=(-3|-2), \vec{a}=\begin{pmatrix}5\\1\end{pmatrix}, \vec{b}=\begin{pmatrix}2\\4\end{pmatrix}, \vec{v}=?$

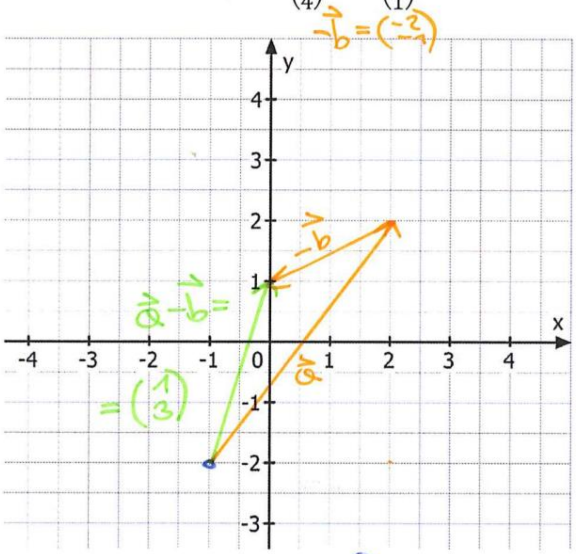


$P=(4|4), \vec{a}=\begin{pmatrix}-8\\-2\end{pmatrix}, \vec{b}=\begin{pmatrix}2\\-5\end{pmatrix}, \vec{v}=?$



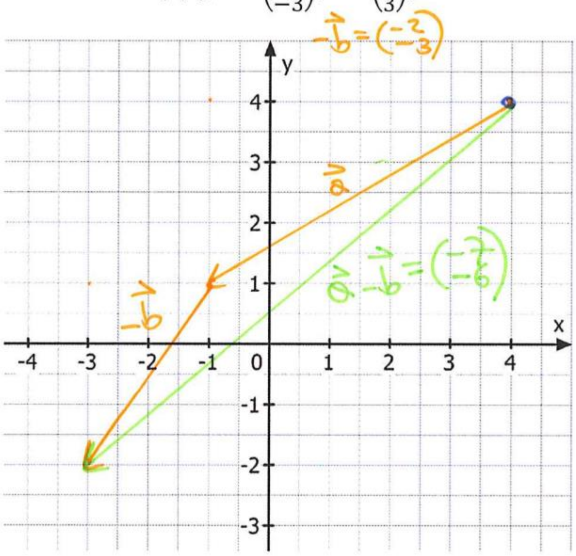
Bsp. 12)

$P=(-1|-2), \vec{a}=\begin{pmatrix}3\\4\end{pmatrix}, \vec{b}=\begin{pmatrix}2\\1\end{pmatrix}, \vec{v}=?$



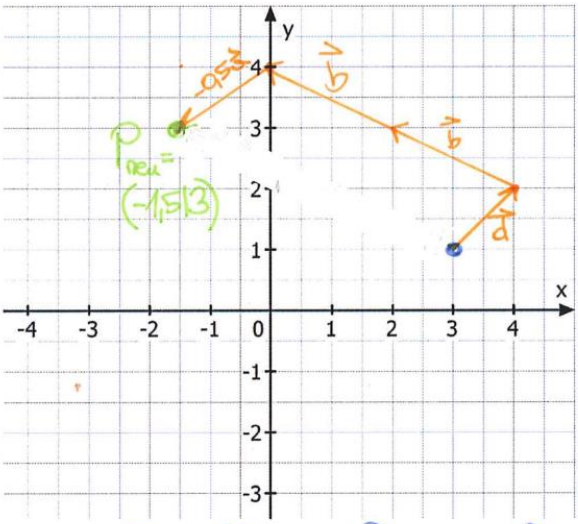
$\vec{a}-\vec{b}=\begin{pmatrix}3\\4\end{pmatrix}-\begin{pmatrix}2\\1\end{pmatrix}=\underline{\underline{\begin{pmatrix}1\\3\end{pmatrix}}}$ ✓

$P=(4|4), \vec{a}=\begin{pmatrix}-5\\-3\end{pmatrix}, \vec{b}=\begin{pmatrix}2\\3\end{pmatrix}, \vec{v}=?$

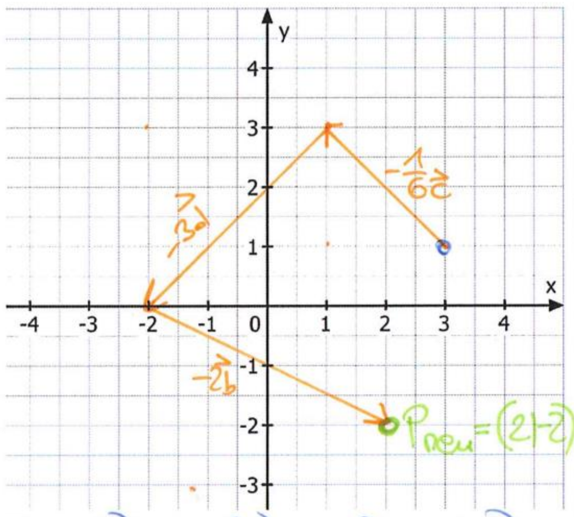


$\vec{a}-\vec{b}=\begin{pmatrix}-5\\-3\end{pmatrix}-\begin{pmatrix}2\\3\end{pmatrix}=\underline{\underline{\begin{pmatrix}-7\\-6\end{pmatrix}}}$ ✓

Bsp. 13)



$$P_{neu} = \begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}1\\1\end{pmatrix} + 2\cdot\begin{pmatrix}-2\\1\end{pmatrix} - 0{,}5\cdot\begin{pmatrix}3\\2\end{pmatrix}$$
$$= \begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}-4\\2\end{pmatrix} + \begin{pmatrix}-1{,}5\\-1\end{pmatrix} =$$
$$= \begin{pmatrix}-1{,}5\\3\end{pmatrix} = \underline{\underline{(-1{,}5|3)}} \checkmark$$



$$P_{neu} = \begin{pmatrix}3\\1\end{pmatrix} - \frac{1}{6}\cdot\begin{pmatrix}12\\-12\end{pmatrix} - 3\cdot\begin{pmatrix}1\\1\end{pmatrix} - 2\cdot\begin{pmatrix}-2\\1\end{pmatrix}$$
$$= \begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}-2\\2\end{pmatrix} + \begin{pmatrix}-3\\-3\end{pmatrix} + \begin{pmatrix}4\\-2\end{pmatrix}$$
$$= \begin{pmatrix}2\\-2\end{pmatrix} = \underline{\underline{(2|-2)}}$$

Bsp. 14)

$\vec{a} = \begin{pmatrix}3\\2\end{pmatrix}, \vec{b} = \begin{pmatrix}6\\4\end{pmatrix}$ $\vec{b} = 2\cdot\vec{a}$ ✓ $\vec{a} \parallel \vec{b}$	$\vec{a} = \begin{pmatrix}13\\24\end{pmatrix}, \vec{b} = \begin{pmatrix}-6{,}5\\12\end{pmatrix}$ $\vec{a} \nparallel \vec{b}$ nicht möglich!	$\vec{a} = \begin{pmatrix}5\\5\end{pmatrix}, \vec{b} = \begin{pmatrix}500\\500\end{pmatrix}$ $\vec{b} = 100\cdot\vec{a}$ ✓ $\vec{a} \parallel \vec{b}$

Bsp. 15)

$\vec{a} = \begin{pmatrix}3\\2\end{pmatrix}, \vec{b} = \begin{pmatrix}b_1\\8\end{pmatrix}$ $\cdot 4$ $\Rightarrow b_1 = 3\cdot 4 = \underline{\underline{12}}$	$\vec{a} = \begin{pmatrix}7\\a_2\end{pmatrix}, \vec{b} = \begin{pmatrix}-3{,}5\\-12\end{pmatrix}$ $:(-2)$ $\cdot(-2)$ $a_2 = (-12)\cdot(-2) = \underline{\underline{24}}$	$\vec{a} = \begin{pmatrix}a_1\\15\end{pmatrix}, \vec{b} = \begin{pmatrix}-3\\30\end{pmatrix}$ $:2$ $a_1 = (-3):2 = \underline{\underline{-1{,}5}}$

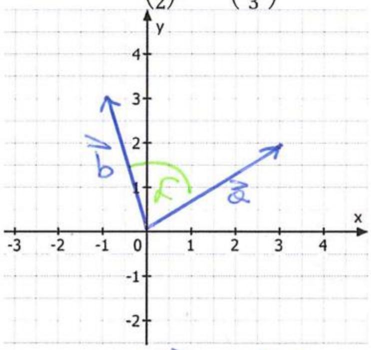
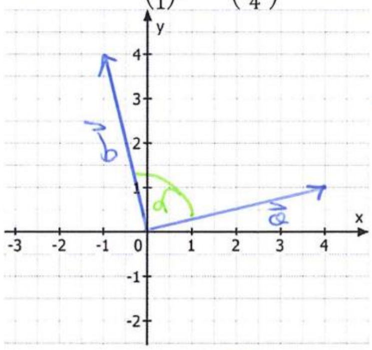
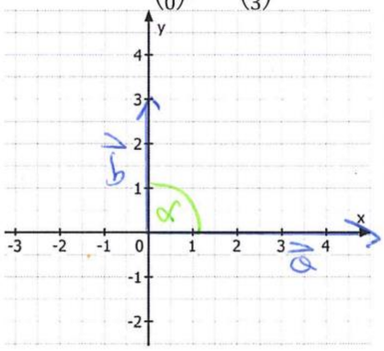
Bsp. 16)

M_{AB}	M_{CE}	M_{BD}
$M_{AB} = \frac{1}{2}(A+B)$ $= \frac{1}{2} \cdot \begin{pmatrix} 9 \\ 0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 4{,}5 \\ 0 \end{pmatrix}}}$	$M_{CE} = \frac{1}{2} \cdot (C+E)$ $= \frac{1}{2} \cdot \begin{pmatrix} -13 \\ -7 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -6{,}5 \\ -3{,}5 \end{pmatrix}}}$	$M_{BD} = \frac{1}{2} \cdot (B+D)$ $= \frac{1}{2} \cdot \begin{pmatrix} 8 \\ -4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 4 \\ -2 \end{pmatrix}}}$

Bsp. 17)

$A = (-5\|-2), B = (7\|2), 1:3$	$A = (3\|4), B = (11\|-4), 1:7$
$T = A + \frac{1}{4} \circ \overrightarrow{AB}$ $T = \begin{pmatrix} -5 \\ -2 \end{pmatrix} + \frac{1}{4} \circ \begin{pmatrix} 12 \\ 4 \end{pmatrix}$ $= \begin{pmatrix} -5 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -2 \\ -1 \end{pmatrix}}}$	$T = A + \frac{1}{8} \circ \overrightarrow{AB}$ $T = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \frac{1}{8} \circ \begin{pmatrix} 8 \\ -8 \end{pmatrix}$ $= \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 4 \\ 3 \end{pmatrix}}}$

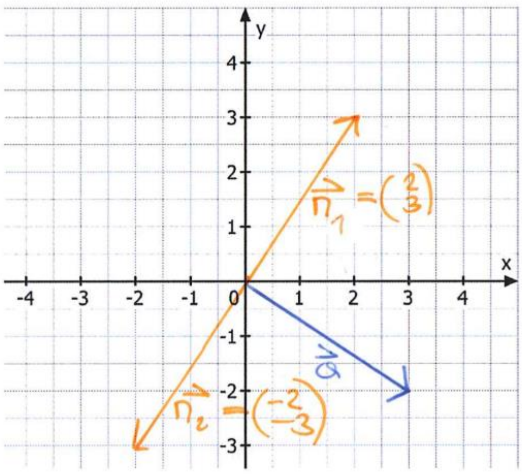
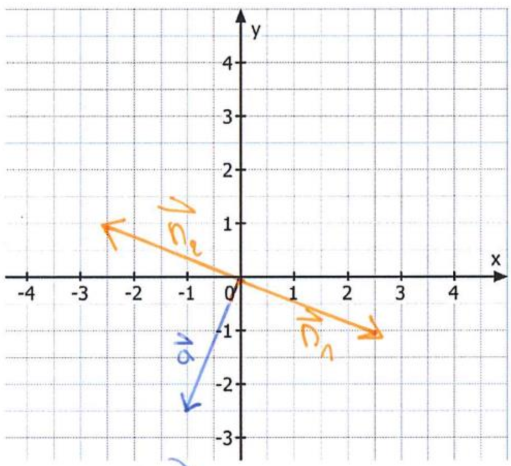
Bsp. 18)

$\vec{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \vec{b} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$	$\vec{a} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \vec{b} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$	$\vec{a} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \vec{b} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$
		
$\cos(\alpha) = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|} = \frac{-3+6}{\sqrt{13} \cdot \sqrt{10}}$ $\Rightarrow \alpha = \cos^{-1}\left(\frac{3}{\sqrt{130}}\right)$ $\underline{\underline{\alpha \approx 74{,}7^\circ}}$	$\alpha = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}\right)$ $\alpha = \cos^{-1}\left(\frac{0}{\sqrt{17} \cdot \sqrt{17}}\right)$ $\underline{\underline{\alpha = 90^\circ}}$	$\alpha = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}\right)$ $\alpha = \cos^{-1}\left(\frac{0}{\sqrt{25} \cdot \sqrt{9}}\right)$ $\underline{\underline{\alpha = 90^\circ}}$

Bsp. 19)

$\vec{a} = \binom{3}{2}$	$\vec{b} = \binom{-3{,}5}{-12}$	$\vec{c} = \binom{-3}{30}$
$\vec{n}_1 = \binom{-2}{3}$ $\vec{n}_2 = \binom{2}{-3}$	$\vec{n}_1 = \binom{12}{-3{,}5}$ $\vec{n}_2 = \binom{-12}{3{,}5}$	$\vec{n}_1 = \binom{-30}{-3}$ $\vec{n}_2 = \binom{30}{3}$

Bsp. 20)

$\vec{a} = \binom{3}{-2}$ – Start im Punkt $P = (0\|0)$	$\vec{a} = \binom{-1}{-2{,}5}$ – Start im Punkt $P = (0\|0)$
 $\vec{n}_1 = \binom{2}{3}$ $\vec{n}_2 = \binom{-2}{-3}$ ✓	 $\vec{n}_1 = \binom{2{,}5}{-1}$ $\vec{n}_2 = \binom{-2{,}5}{1}$ ✓

Bsp. 21)

$\vec{a} = \binom{3}{2}, \vec{b} = \binom{6}{-10}$	$\vec{a} = \binom{7}{2}, \vec{b} = \binom{-6}{21}$	$\vec{a} = \binom{100}{-15}, \vec{b} = \binom{-3}{-20}$
$\vec{a} \cdot \vec{b} = \binom{3}{2} \cdot \binom{6}{-10}$ $= 18 - 20 = -2 \neq 0$ NEIN! $\vec{a} \not\perp \vec{b}$	$\binom{7}{2} \cdot \binom{-6}{21} = -42 + 42 = 0$ $\vec{a} \perp \vec{b}$	$\binom{100}{-15} \circ \binom{-3}{-20} =$ $= -300 + 300 = 0$ $\vec{a} \perp \vec{b}$

Bsp. 22)

$\vec{a} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}, \vec{b} = \begin{pmatrix} b_1 \\ 8 \end{pmatrix}$	$\vec{a} = \begin{pmatrix} 5 \\ a_2 \end{pmatrix}, \vec{b} = \begin{pmatrix} 20 \\ -10 \end{pmatrix}$	$\vec{a} = \begin{pmatrix} a_1 \\ -8 \end{pmatrix}, \vec{b} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$
$\vec{a} \cdot \vec{b} = 0$ $\begin{pmatrix} -4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ 8 \end{pmatrix} = 0$ $-4b_1 + 16 = 0 \mid -16$ $-4b_1 = -16 \mid :(-4)$ $\underline{\underline{b_1 = 4}}$	$\vec{a} \cdot \vec{b} = 0$ $\begin{pmatrix} 5 \\ a_2 \end{pmatrix} \cdot \begin{pmatrix} 20 \\ -10 \end{pmatrix} = 0$ $100 - 10a_2 = 0 \mid -100$ $-10a_2 = -100 \mid :(-10)$ $\underline{\underline{a_2 = 10}}$	$\vec{a} \cdot \vec{b} = 0$ $-2 \cdot a_1 - 64 = 0 \mid +64$ $-2a_1 = 64 \mid :(-2)$ $\underline{\underline{a_1 = -32}}$