LÖSUNGEN FLÄCHENINHALTE

Bsp. 1) Berechne den Inhalt jener Fläche, der vom Graphen von f und der x-Achse im gegebenen Intervall eingeschlossen wird. Mach dir eine Skizze.

a. $f(x) = 3x^2 - 3$ $[-2; 3]$

$f(x)=0 \iff x_1=-1,\ x_2=1$

$A = \int_{-2}^{-1} f(x)\,dx - \int_{-1}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$

$A = 4-(-4)+20 = \underline{\underline{28\,E^2}}$

b. $f(x) = 2x + 4$ $[-6; 2]$

$f(x)=0 \iff x=-2$

$A = -\int_{-6}^{-2} f(x)\,dx + \int_{-2}^{2} f(x)\,dx$

$A = -(-16)+16 = \underline{\underline{32\,E^2}}$

c. $f(x) = x^3 - 9x$ $[-5; 4]$

$f(x)=0 \iff x_1=-3,\ x_2=0,\ x_3=3$

$A = -\int_{-5}^{-3} f(x)\,dx + \int_{-3}^{0} f(x)\,dx - \int_{0}^{3} f(x)\,dx + \int_{3}^{4} f(x)\,dx =$

$A = -(-64)+20{,}25-(-20{,}25)+12{,}25 = \underline{\underline{116{,}75\,E^2}}$

d. $f(x) = x^4 - 4x^2$ $[-1; 3]$

$f(x)=0 \iff x_1=-2,\ x_2=0,\ x_3=2$

$A = -\int_{-1}^{0} f(x)\,dx - \int_{0}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx$

$A = -(-1{,}13)-(-4{,}27)+16{,}87 = \underline{\underline{22{,}27\,E^2}}$

e. $f(x) = e^x - 2$ $[-3; 5]$

$f(x)=0 \iff x=0{,}69$

$A = -\int_{-3}^{0{,}69} f(x)\,dx + \int_{0{,}69}^{5} f(x)\,dx$

$A = 5{,}44 + 137{,}8 = \underline{\underline{143{,}24\,E^2}}$

f. $f(x) = \frac{1}{x} - 1$ $[0{,}5; 3]$

$f(x)=0 \iff x=1$

$A = \int_{0{,}5}^{1} f(x)\,dx - \int_{1}^{3} f(x)\,dx$

$A = 0{,}19 + 0{,}9 = \underline{\underline{1{,}09\,E^2}}$

Bsp. 2) Eine Funktion f ist gegeben. Bestimme den Parameter e so, dass der Graph von f in $[1; e]$ mit $e > 1$ den Flächeninhalt A mit der x-Achse einschließt.

a. $f(x) = -x + 3$ $A = 10\,E^2$ -2 $\forall\ 8E^2$

$f(x)=0 \iff x=3$

① $\int_{1}^{3} f(x)\,dx = \underline{\underline{2}}$

② $\int_{3}^{e} f(x)\,dx = -8$

$\left(-\frac{x^2}{2}+3x\right)\Big|_3^e = -8$

$\left(-\frac{e^2}{2}+3e\right) - \left(-\frac{9}{2}+9\right) = -8$

GG $(e_1=-1)$ $\underline{\underline{e_2=7}}$ $\boxed{[1;7]}$

b. $f(x) = -3x^2 + 12$ $A = 37\,E^2$ -5 $\forall\ 32E^2$

$f(x)=0 \iff (x_1=-2)$ $x_2=2$ ✓

① $\int_{1}^{2} f(x)\,dx = 5$

② $\int_{2}^{e} f(x)\,dx = -32$ $-32!$

GG $(e_1=-2)$ $e_2=4$ ✓ $\boxed{[1;4]}$

c. $f(x) = x^3 - 3x^2$ $A = 270\,E^2$ -6

$f(x)=0 \iff (x_1=0)$ $x_2=3$

① $\int_{1}^{3} f(x)\,dx = \underline{\underline{-6}} \to (+6)$ $\forall\ 264E^2$

② $\int_{3}^{e} f(x)\,dx = 264$

$(e_1\approx-4{,}88)$ $\underline{\underline{e_2=7}}$ $\underline{\underline{[1;7]}}$

d. $f(x) = -4x + 16$ $A = 146\,E^2$ -18 $\forall\ 128E^2$

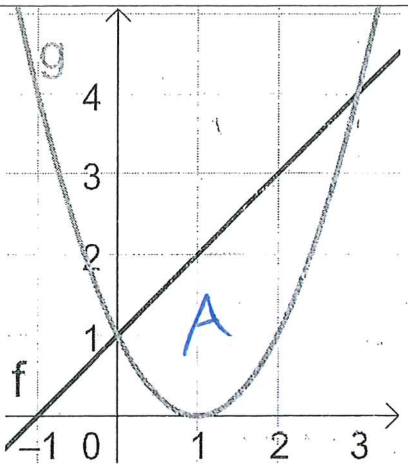
$f(x)=0 \iff x=4$

① $\int_{1}^{4} f(x)\,dx = \underline{\underline{18}}$

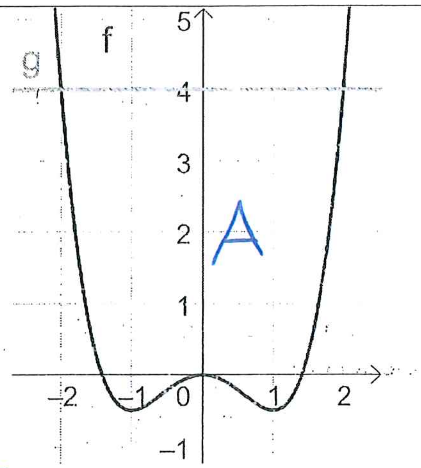
② $\int_{4}^{e} f(x)\,dx = -128$

$(e_1=-4)$ $\underline{\underline{e_2=12}}$ $\underline{\underline{[1;12]}}$

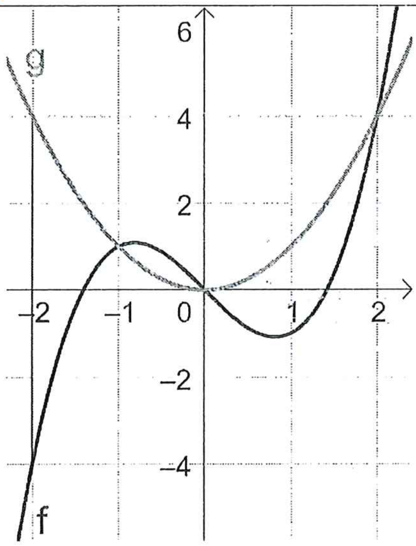
Bsp. 3) Gib mit den Funktionen $f(x)$, $g(x)$, $h(x)$ eine Formel zur Berechnung der markierten Fläche an.



$$A=\int_0^3 [f(x)-g(x)]\,dx$$

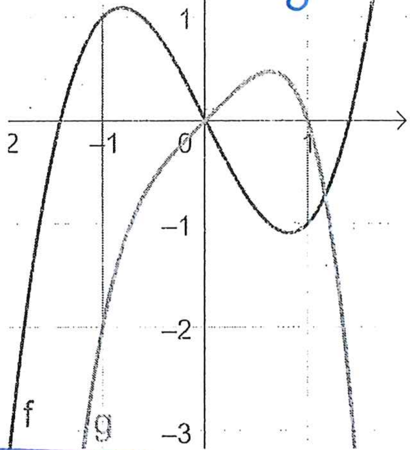


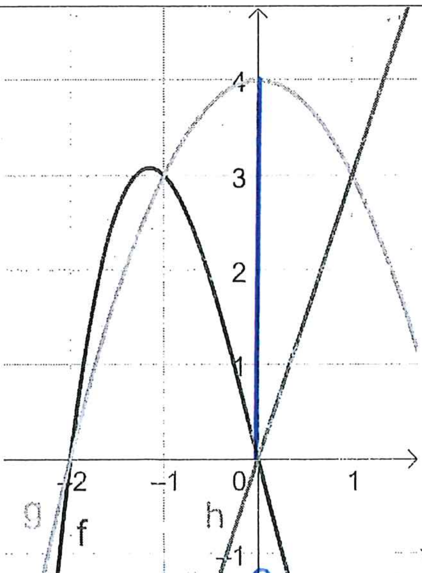
$$A=\int_{-2}^{2} [g(x)-f(x)]\,dx$$



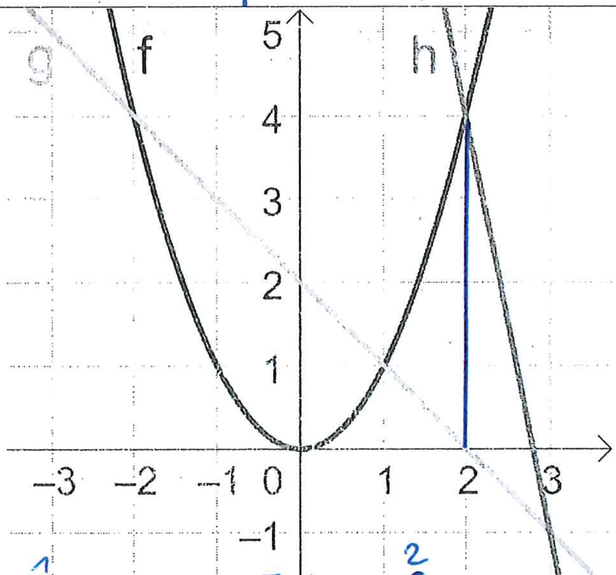
$$A=\int_{-2}^{-1} [g(x)-f(x)]\,dx+\int_{-1}^{0} [f(x)-g(x)]\,dx+\int_{0}^{2} [g(x)-f(x)]\,dx$$

$$A=\int_{-1}^{0} [f(x)-g(x)]\,dx+\int_{0}^{1} [g(x)-f(x)]\,dx$$





$$A=\int_{-2}^{-1} [f(x)-g(x)]\,dx+\int_{-1}^{0} [g(x)-f(x)]\,dx + \int_{0}^{1} [g(x)-h(x)]\,dx$$



$$A=\int_{-2}^{1} [g(x)-f(x)]\,dx+\int_{1}^{2} [f(x)-g(x)]\,dx + \int_{2}^{3} [h(x)-g(x)]\,dx$$

Bsp. 4) Berechne den Flächeninhalt ohne Technologie, der von den Graphen der Funktionen f und g begrenzt wird.

a. $f(x) = x$, $g(x) = -x^2 + 2$ ① $f(x) = g(x)$ $x = -x^2 + 2 \quad | +x^2, -2$ $x^2 + x - 2 = 0$ $x_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + 2} \quad | \frac{3}{2}$ $x_1 = -2 \quad x_2 = 1$ ② $g(0) = 2$ $f(0) = 0$ $g(x) > f(x) \quad [-2;1]$ ③ $\int_{-2}^{1} [g(x) - f(x)]dx = \int_{-2}^{1} [-x^2 + 2 - x]dx =$ $-\frac{x^3}{3} + 2x - \frac{x^2}{2} \Big|_{-2}^{1} = 4{,}5E^2$	**b.** $f(x) = x^2 - 1$, $g(x) = -x^2 + 1$ ① $f(x) = g(x)$ $x_1 = -1; \; x_2 = 1$ ② $\int_{-1}^{1} [g(x) - f(x)]dx$ $= 2{,}67E^2$
c. $f(x) = 3x^3 - 2x^2 + 1$, $g(x) = 3x^3 + 2x^2 - 3$ ① $f(x) = g(x)$ $x_1 = -1 \quad x_2 = 1$ ② $A = \int_{-1}^{1} [f(x) - g(x)]dx$ $= 5{,}33E^2$	**d.** $f(x) = x^2 + 2x - 1$, $g(x) = -x - 1$ ① $f(x) = g(x)$ $x_1 = -3; \; x_2 = 0$ ② $A = \int_{-3}^{0} [g(x) - f(x)]dx$ $= 4{,}5E^2$

Bsp. 5) Berechne den Flächeninhalt, der von den Graphen der Funktionen f und g begrenzt wird.

a. $f(x) = x^4 - 3x^2 + 4$, $g(x) = 2x + 4$ ① $f(x) = g(x) \Leftrightarrow x_1 = -1, x_2 = 0; \; x_3 = 2$ ② $A = \int_{-1}^{0} [f(x) - g(x)]dx + \int_{0}^{2} [g(x) - f(x)]dx$ $= 0{,}2 + 5{,}6 = 5{,}8E^2$	**b.** $f(x) = -x$, $g(x) = x^3 - 2x$ ① $f(x) = g(x): \; x_1 = -1; \; x_2 = 0; \; x_3 = 1$ ② $A = \int_{-1}^{0} [g(x) - f(x)]dx + \int_{0}^{1} [f(x) - g(x)]dx$ $A = 0{,}25 + 0{,}25 = 0{,}5E^2$
c. $f(x) = x^3 + 4x^2 + 2$, $g(x) = 8 - x$ ① $f(x) = g(x): x_1 = -3; \; x_2 = -2; \; x_3 = 1$ ② $A = \int_{-3}^{-2} [f(x) - g(x)]dx + \int_{-2}^{1} [g(x) - f(x)]dx$ $A = 0{,}58 + 11{,}25 = 11{,}83E^2$	**d.** $f(x) = x^3 - 4x$, $g(x) = 4 - x^2$ ① $f(x) = g(x): \; x_1 = -2; \; x_2 = -1; \; x_3 = 2$ ② $A = \int_{-2}^{-1} [f(x) - g(x)]dx + \int_{-1}^{2} [g(x) - f(x)]dx$ $A = 0{,}58 + 11{,}25 = 11{,}83E^2$

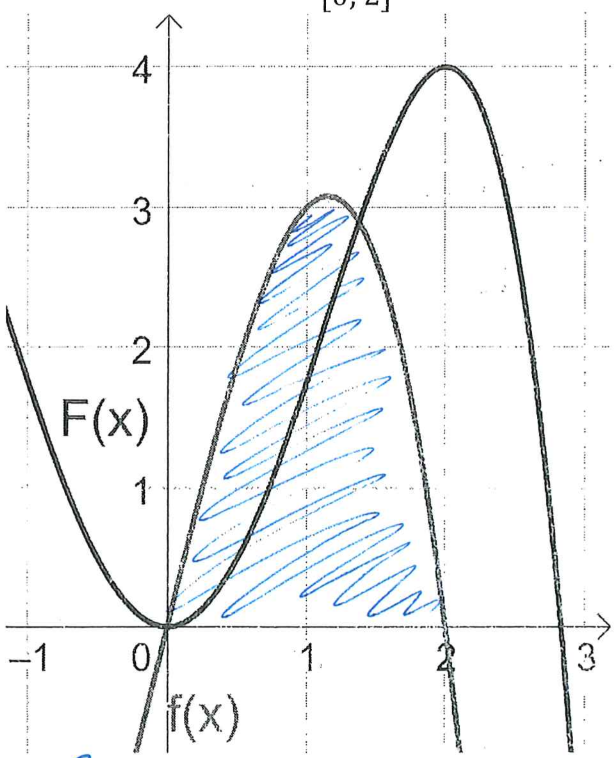
Bsp. 6) Berechne den Flächeninhalt, der vom Graphen der Funktion f, ihrer Tangente im angegebenen Punkt P sowie der y-Achse eingeschlossen wird.

Video 10

a. $f(x) = x^2 - 5x + 2$, $P = (4\vert y_P)$	**b.** $f(x) = -x^3 + 4x^2$, $P = (1\vert y_P)$
① $f(4) = -2$; $t: y = 3x + d$ $f'(4) = 3$ ↗ Punkt: $-2 = 12 + d \Rightarrow d = -14$ $t: y = 3x - 14 \Rightarrow t(x) = 3x - 14$ ② $A = \int_0^4 [f(x) - t(x)]dx = \underline{21{,}33 E^2}$	① $f(1) = 3$ ↗ $t(x) = 5x + d$ $f'(1) = 5$ $\quad 3 = 5 + d \Rightarrow d = -2$ $\underline{t(x) = 5x - 2}$ ② $A = \int_0^1 [f(x) - t(x)]dx = \underline{0{,}58 E^2}$
c. $f(x) = x^4 - 2x^2$, $P = (2\vert y_P)$	**d.** $f(x) = -x^2 + 4x + 8$, $P = (5\vert y_P)$
① $f(2) = 8$ ↗ $t(x) = 24x + d$ $f'(2) = 24$ $\quad 8 = 48 + d \Rightarrow d = -40$ $\underline{t(x) = 24x - 40}$ ② $A = \int_0^2 [f(x) - t(x)]dx = \underline{33{,}07 E^2}$	① $f(5) = 3$ ↗ $t(x) = -6x + d$ $f'(5) = -6$ $\quad 3 = -30 + d \Rightarrow d = 33$ $\underline{t(x) = -6x + 33}$ ② $A = \int_0^5 [t(x) - f(x)]dx = \underline{41{,}67 E^2}$

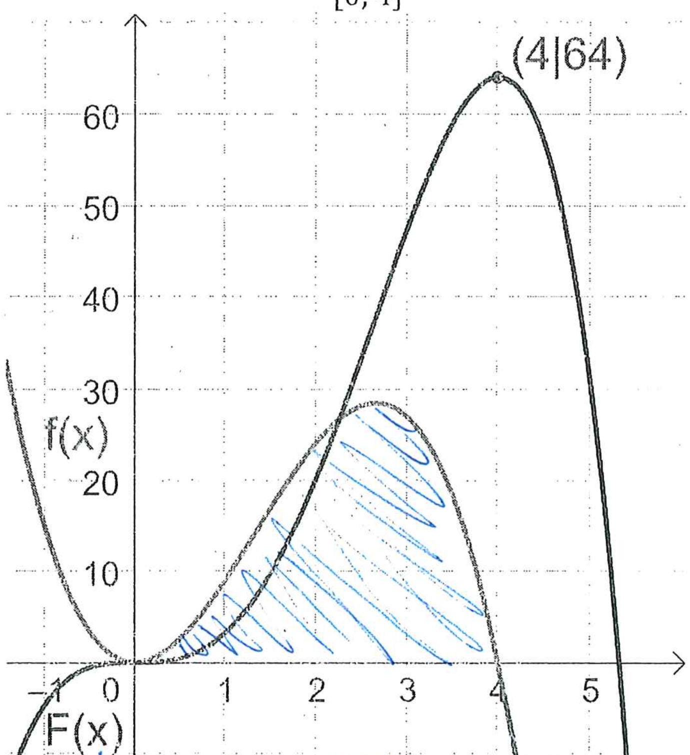
Bsp. 7) Gegeben sind ein Graph einer Polynomfunktion f und der Graph einer ihrer Stammfunktionen F. Der Graph von f und die positive x-Achse begrenzen im gegebenen Intervall ein endliches Flächenstück. Markiere und ermittle den Flächeninhalt dieses Flächenstücks.

[0; 2]



$A = \int_0^2 f(x)dx = F(2) - F(0)$
$= 4 - 0 = \underline{4 E^2}$

[0; 4]



$A = \int_0^4 f(x)dx = F(4) - F(0)$
$= 64 - 0 = \underline{64 E^2}$